# The Information Content of Revealed Beliefs in Portfolio Holdings

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**Abstract**

In this paper, we elicit heterogeneous fund manager beliefs on expected stock returns from funds' portfolio holdings at each quarter-end. Revealed beliefs are extracted by assuming that each fund manager aims to outperform a certain benchmark portfolio by choosing an optimal risk-return tradeoff. We then construct a measure of *differences* in beliefs among fund manages for each stock, the belief difference index (BDI). Specifically, we categorize funds into two groups, those with beliefs highly correlated with realized stock returns and those with beliefs less correlated. We then compute BDI as the difference in the average beliefs between these two groups. Sorting stocks based on BDI, we find that the annualized return difference between the top and bottom decile is about two to five percent. The predicability of BDI significantly weakens for extremely small or large stocks, or when risk among stock returns is modeled using an identity or a diagonal matrix. These results indicate that 1) fund managers do adjust for risk when making portfolio decisions; 2) risk-return optimization is less used for small stocks by fund managers; and 3) there are less disagreements among managers about large stock returns.

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# 1 Introduction

Portfolio theory is a cornerstone of modern finance. Pioneered by the work of Markowitz (1952), it has led to a vast amount of research exploring optimal portfolios under various constraints (e.g., short-sales), frictions (e.g., heterogeneous information) and computational limits (e.g., estimation of the variance-covariance matrix). This has resulted in a set of recipes for converting portfolio theory's raw ingredients—beliefs on the structure of stock returns—into its finished product, the optimal portfolio. In contrast, little attention has been devoted to the dual problem: extracting beliefs about the structure of expected stock returns from observed portfolio holdings. In this paper, we focus on the information embedded in the cross-sectional portfolio holdings of mutual fund managers, particularly the revealed heterogeneous fund manager beliefs about expected stock returns. We examine whether these revealed beliefs contain information about the skills of mutual fund managers and/or how they are embedded into the prices of common stocks.

To elicit fund managers' beliefs we make three assumptions. First, we assume that mutual fund managers possess heterogeneous beliefs. Second, we assume that each fund manager has a benchmark index. He wishes to outperform his benchmark with the minimum amount of risk subject to a performance target. This objective function is the same as that discussed in Roll (1992) and is commonly observed in practice. Last, we assume that the variance-covariance matrix of asset returns is common knowledge among investors. This assumption is motivated by the empirical finding that estimating the second moment of a return-generating processes from historical data is considerably easier than estimating the first moment. The widely-implemented Black-Litterman model (1992) also adopts this assumption.

In our model, a mutual fund manager's portfolio holdings are the outcome of an optimization based on his beliefs about stock returns (which are specific to him) and about the variance-covariance structure of these returns (which is common across investors). Therefore, his beliefs about stock returns can be easily backed out if the variance-covariance structure is known. Empirically, we estimate the variance-covariance matrix based on historical return data, which are observable to all investors. In estimating the variance-covariance matrix, we use a multi-factor model. We motivate this by noting that multi-factor models are commonly used in the money management industry.

After backing out these revealed beliefs, we construct a measure of fund managers' stock picking ability by correlating each manager's revealed beliefs about stock returns with the subsequently realized returns. By construction, this correlation is not affected by investor-specific characteristics such as heterogenous risk-aversion and ensures that we capture the effect of heterogenous revealed beliefs. We then measure the *differences* in beliefs between the top thirty percent of fund managers ranked by this correlation and all the remaining fund managers, the belief difference index (BDI). We conjecture that ex post returns are more consistent with the beliefs of the best managers than with the beliefs of all other managers and the current prices are more consistent with the beliefs of the majority: the bottom seventy percent. Hence the differences in beliefs between these two groups of managers reveal information not embedded in the stock price: A large positive BDI statistic indicates that the positive information is not embedded into the stock price while a large negative BDI statistic suggests that the negative information is not embedded into the stock price. We sort stocks into deciles according to BDI and examine the subsequent three-month performance across the decile portfolios. The results show that, on average, stocks with higher BDI statistics outperform stocks with lower BDI statistics, indicating that revealed beliefs contain valuable information about future stock returns. We find the annualized performance spread between the top and bottom decile funds is about two to five percent, which is significant, both economically and statistically. These performance differences are not explained by variations in risk or style factors.

We also sort stocks into three groups: small, medium, and big according to their size at the end of each quarter and redo our analysis. We find that the significant performance difference between the top and bottom BDI deciles comes from the medium size group. This result suggests that the stock picking skills of fund managers are reflected mostly in medium size stocks.

Interestingly, when we replace the variance-covariance matrix used in estimating revealed beliefs with an identity matrix or a diagonal matrix (that is, ignoring the idiosyncratic or the systematic risk in stock returns), the result on the BDI predicability is weaker. That is, by taking into account of the fact that fund managers believe stock returns exhibit risk and this risk is captured by idiosyncratic as well systematic components, the information content embedded in the cross-sectional portfolio holdings is sharper. This finding suggests that fund managers do care about risk when making portfolio decisions.

It is important to know whether there is information in fund holdings, in part because this information allows us to make some inferences about the degree to which the equity market is informationally efficient. One of the most frequently cited arguments for efficiency is the apparent lack of ability of mutual fund managers. However, Berk and Green (2004) show that managerial ability is consistent with a lack of performance persistence in equilibrium. Therefore, assessing managerial ability requires more powerful techniques than those which simply analyze historical fund returns. Our technique shows that many managers are able to forecast returns, that is, they possess stock picking abilities.[1]

Recently, there have been various attempts to investigate the information revealed by portfolio holdings for performance evaluation of portfolio managers. Grinblatt and Titman (1989); Daniel, Grinblatt, Titman, and Wermers (1997); Graham and Harvey (1996); Wermers (2000); Chen, Jegadeesh, and Wermers (2000); Ferson and Khang (2002); Cohen, Coval, and Pastor (2005); Kacperczyk, Sialm, and Zheng (2005); Kacperczyk, Sialm, and Zheng (2008); Cremers and Petajisto (2006); Kacperczyk and Seru (2007) and Breon-Drish and Sagi (2008) have made contributions along this line. Instead of future fund performance, our study extends this line of research by focusing on the implication of information revealed in cross-sectional portfolio holdings for future stock returns.

There have also been attempts to look beyond the information revealed in historical stock return data for future stock returns. Lo and Wang (2000; 2001) find that turnover satisfies an approximately linear $k$-factor structure and Goetzmann and Massa (2006) identify factors through a sample of net flows to nearly 1000 U.S. mutual funds over a year and a half period. Factors embedded in flow and turn-over data are shown to have valuable information for pricing stocks. Chen, Jegadeesh, and Wermers (2000) find that the consensus opinion of mutual industry (that is, the aggregate active trade of the mutual fund industry) reflects relative superior information about the value of the stock. Wermers, Yao, and Zhao (2007) find that stocks held by top ranked funds (according to measures such as Cohen, Coval, and Pastor (2005)) outperform the rest on average, indicating the investment value of mutual funds. Cohen, Polk, and Silli (2009) also find that top five stocks held by actively managed funds tend to outperform the market. Our paper is closely related to this line of research. Our paper complements the existing literature by formally proposing a method to extract the information embedded in the cross-sectional portfolio holding for fund managers' beliefs and study how the dispersion of these revealed beliefs (opinions) is related to the inefficiency of the market and future stock returns.

The remainder of this paper is organized as follows. In Section 2, we present our method-

[1] However, whether a manager can outperform the market also depends on whether he has superior market timing abilities, which this study is silent about.

ology for extracting beliefs from portfolio holdings. Section 3 provides the definition of BDI. Section 4 describes the data used and the empirical implementation of the model. In Section 5, we construct BDI empirically and evaluate whether BDI has valuable information for predicting future stock returns. We conclude in Section 6.

# 2 Eliciting Fund Managers' Heterogeneous Beliefs

In this section we present a simple portfolio optimization model to highlight the theoretical foundation for eliciting portfolio managers' heterogeneous beliefs. The objective of this section is to demonstrate how one can back out heterogeneous beliefs about future excess returns from observed portfolio holdings. To do so requires an assumption about the behavior of fund managers. We assume that a fund manager is evaluated relative to some passive benchmark portfolio. Compared with the standard textbook portfolio problem where the investor seeks to minimize return volatility for a given level of expected return, the fund manager in this setup seeks to minimize tracking error volatility for a given level of return *in excess of the benchmark return*. In other words, a fund manager is indifferent to the whims of his benchmark, as long as he can outperform it. As Roll (1992) points out, managers who implement this optimization program do not hold mean-variance efficient portfolios. However, it is clear that tracking error criteria are widely used in practice. Given this, it is reasonable to assume that managers implement such an optimization program. In what follows, we first detail the return-generating process for risky and risk-free assets and the information structure among the fund managers. We then solve the fund manager's portfolio optimization problem. Then, we show how a fund manager's beliefs about stock returns can be identified up to a constant given his (optimal) portfolio holdings and benchmark.

To develop the model, we first focus on the managers' optimization problem. In this problem the investment opportunity set consist of a risk-free asset with a constant return, $r_f$, and $n$ risky assets where the $i$th asset's *excess return* over the risk-free rate $r_f$ is denoted as $\tilde{r}_i$ and the excess returns of the $n$ risky assets can be written as $\tilde{\mathbf{r}} = [\tilde{r}_1, ..., \tilde{r}_n]'$, a $n \times 1$ vector. The $n$ risky assets have the following full rank variance-covariance matrix:

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma^2_{1,1} & \cdots & \sigma^2_{1,n} \\ \vdots & \ddots & \vdots \\ \sigma^2_{n,1} & \cdots & \sigma^2_{n,n} \end{bmatrix}, \tag{1}$$

We assume that there are $m$ mutual fund managers in this economy. Fund managers possess a common knowledge of $\mathbf{\Sigma}$, but are heterogeneously informed about the risky assets' excess returns. We use $\mu_{mi}$ to denote manager $m$'s belief of asset $i$'s expected excess return. Manager $m$'s belief on the $n$ assets' expected excess returns can be written as $\mu_m = [\mu_{m1}, \ldots, \mu_{mn}]'$, and the beliefs of all the managers can be written as $\mu = [\mu_1, \ldots, \mu_m]'$, which is an $m \times n$ matrix.[2]

Let $w_{m0}$ denote the percentage of wealth (or portfolio weight) invested by manager $m$ in the risk-free asset and let $\mathbf{w}_m = [w_{m1}, \ldots, w_{mn}]'$ denote the vector of portfolio weights in each of the $n$ risky assets. The portfolio weights satisfy the following budget constraint:

$$w_{m0} + \mathbf{1}'\mathbf{w}_m = 1. \tag{2}$$

Each manager has a benchmark portfolio (against which he will be judged), and we denote by $\mathbf{q}_m = [q_{m1}, \ldots, q_{mn}]'$ the vector of manager $m$'s benchmark portfolio weights in each of the $n$ risky assets. We assume that the benchmark consists only of risky assets; consequently, benchmark weights satisfy the following constraint:

$$\mathbf{1}'\mathbf{q}_m = 1. \tag{3}$$

Fund managers choose portfolio weights to maximize the expected return over the benchmark (i.e., active return) and minimize the tracking error volatility (i.e., active risk). We denote manager $m$'s active return by $\tilde{z}_m$, where:

$$\tilde{z}_m = (\mathbf{w}_m - \mathbf{q}_m)'(\tilde{\mathbf{r}} + \mathbf{1}r_0) + w_{m0}r_0 \tag{4}$$

$$= (\mathbf{w}_m - \mathbf{q}_m)'\tilde{\mathbf{r}}. \tag{5}$$

Then, his expected active return is $E[\tilde{z}_m] = (\mathbf{w}_m - \mathbf{q}_m)'\mu_m$ and his active risk is $Var[\tilde{z}_m] = (\mathbf{w}_m - \mathbf{q}_m)'\mathbf{\Sigma}(\mathbf{w}_m - \mathbf{q}_m)$.

In other words, fund manager $m$, conditional on his beliefs, $\mu_m$, his benchmark, $\mathbf{q}_m$, and his desired level of expected active return, $E[\tilde{z}_m]$, chooses his portfolio weights, $\mathbf{w}_m$, so as to minimize active risk (tracking error volatility):

$$\mathbf{w}_m^* = \underset{\{\mathbf{w}_m, w_{0m}\}}{\operatorname{argmin}} \{(\mathbf{w}_m - \mathbf{q}_m)'\mathbf{\Sigma}(\mathbf{w}_m - \mathbf{q}_m)\}, \tag{6}$$

subject to the budget constraint (2) and

$$E[\tilde{z}_m] = (\mathbf{w}_m - \mathbf{q}_m)'\mu_m. \tag{7}$$

[2]Unless otherwise noted, we use boldface letters to denote vectors or matrices.

To solve, we construct the Lagrangian

$$L = (\mathbf{w}_m - \mathbf{q}_m)'\mathbf{\Sigma}(\mathbf{w}_m - \mathbf{q}_m) + \lambda_m(E[\tilde{z}_m] - (\mathbf{w}_m - \mathbf{q}_m)'\mu_m), \tag{8}$$

where $\lambda_m$ is the Lagrangian multiplier for fund manager $m$. Differentiating with respect to $(\mathbf{w}_m - \mathbf{q}_m)$ obtains the first order condition:

$$(\mathbf{w}^*_m - \mathbf{q}_m) = \lambda_m \mathbf{\Sigma}^{-1}\mu_m. \tag{9}$$

It follows from the budget constraint (2), that the portfolio share allocated to the risk free asset is:

$$w^*_{0m} = -\lambda_m \mathbf{1}'\mathbf{\Sigma}^{-1}\mu_m. \tag{10}$$

Rearranging the terms in the first order condition (9), we obtain an expression for the beliefs of fund manager $m$:

$$\mu_m = \lambda_m^{-1}\mathbf{\Sigma}(\mathbf{w}^*_m - \mathbf{q}_m), \tag{11}$$

where the only unsolved term is $\lambda_m$.

Combining the first order condition (9) and the active return constraint (7), we obtain an expression for the Lagrange multiplier $\lambda_m$:

$$\lambda_m = \frac{(\mathbf{w}^*_m - \mathbf{q}_m)'\mathbf{\Sigma}(\mathbf{w}^*_m - \mathbf{q}_m)}{E[\tilde{z}_m]} = \frac{Var[\tilde{z}_m]}{E[\tilde{z}_m]}, \tag{12}$$

which is a constant specific to each fund manager.[3]

Thus, given an estimate of $\mathbf{\Sigma}$, denoted by $\hat{\mathbf{\Sigma}}$, a fund manager's private beliefs, $\mu_m$ can be immediately revealed (up to a multiplicative constant, $\lambda_m$) by $(\mathbf{w}^*_m - \mathbf{q}_m)$. Since the variance-covariance matrix can be estimated from historical return data, this leads to the extraction of heterogeneous beliefs from portfolio holdings. This finding is summarized in the following result.

**Result 1** *For a given investor's portfolio weights $\mathbf{w}_m$, benchmark portfolio $\mathbf{q}_m$, and estimated variance-covariance matrix $\hat{\mathbf{\Sigma}}$, the investor's private beliefs on expected returns, $\mu_m$, are revealed up to a constant:*

$$\hat{\mu}_m = \lambda_m^{-1}\hat{\mathbf{\Sigma}}(\mathbf{w}_m - \mathbf{q}_m). \tag{13}$$

*where $\hat{\mu}_m$ denotes an estimate of $\mu_m$ and $\lambda_m$ is an investor-specific constant.*

[3] Our framework is similar to that in Black and Litterman (1992) except that they impose one more constraint, which is that the market portfolio reflects equilibrium beliefs of expected returns.

We term these sets of beliefs "*revealed beliefs*" because they are revealed by portfolio holdings. These revealed beliefs have several unique properties. First, they are forward-looking. That is, estimated at one point in time, they are expectations about future stock returns from that point on. Second, they are heterogeneous across fund managers. By choosing different optimal portfolios at the same time, fund managers reveal their differences in opinion. Third, they are inherently dynamic. As portfolio choices may change over time for the same fund manager, his revealed beliefs also vary over time.

# 3 Evaluating the Information Content of Revealed Beliefs

Since *revealed beliefs* capture a fund manager's *ex-ante* beliefs on future stock returns, how closely these beliefs match the *ex-post* returns is an intuitive measure of the manager's stock picking ability.[4] Specifically, to capture this ability, we use the correlation between the realized excess returns and the revealed beliefs about excess returns (as computed in Result 1) using all stocks in fund $m$'s portfolio. Note that Result 1) shows that the revealed belief $\mu_m$ is identified up to a multiplicative constant, $\lambda_m$, which could be related to a number of manager specific attributes such as heterogenous risk aversion, varying desired level of expected active returns, etc. However, this constant is normalized in the computation of correlation. Hence, by construction, the correlation between realized returns and revealed beliefs are not affected by manager-specific parameter $\lambda_m$.

Next we categorize managers into two categories: "informed" and "less informed" based this correlation. Informed managers are those who are ranked in the top thirty percent of all funds based on correlations of their revealed beliefs with realized stock returns. That is, informed managers are those with better stock picking abilities. We consider all managers in the bottom seventy percent of the correlation distribution to be less informed. We hypothesize that our informed managers have more accurate beliefs about ex post stock returns than their less informed peers. We assume that current prices are consistent with the beliefs of the

[4]Of course, this depends on the portfolio holdings being the interior solution to the optimization program just described. In practice, the portfolio holdings may be a corner solution, a solution to a completely different optimization program (e.g. traditional mean-variance optimization), or a solution to a much more complicated program (e.g. one involving the aerodynamics of darts). In any of these cases the relation will obviously not hold. However, it will become clear that this can only weaken our results.

less informed fund managers since they are in majority. This characterization is consistent with an equilibrium of the type proposed by Grossman and Stiglitz (1980), where the cost of obtaining information deters a certain fraction of investors from obtaining it. Since the less-informed investors' beliefs simply reflect current market prices, the difference between the informed investors' beliefs and those of the less-informed investors measures how much information held by informed investors is yet to be embedded into the stock price.

For any stock, the difference between the informed and less-informed beliefs constitute a measure of information content which should predict future non-systematic price movement, as the information of the informed investors is eventually incorporated into the price.[5] Hence the differences in beliefs between these two groups of managers reveal information not embedded in the stock price: A large positive difference indicates that the positive information is not embedded into the stock price while a large negative difference suggests that the negative information is not embedded into the stock price. We refer to this measure as the "*belief difference index*" or BDI.

**Definition 1** *For a given $(m \times n)$ matrix of portfolio revealed beliefs $\hat{\mu}$, a $n$-vector of ex-post excess returns $\mathbf{r}$, and an $m$-vector of correlations between revealed beliefs and realized beliefs, $\hat{\mathbf{c}} = [\hat{cor}(\hat{\mu}_0', \mathbf{r}), \ldots, \hat{cor}(\hat{\mu}_m', \mathbf{r})]'$ where $\hat{\mu}_m$ is the $m$th row vector of $\hat{\mu}$ matrix and $\hat{cor}(\cdot)$ is the sample correlation function, the BDI is an $n$-vector defined as:*

$$BDI \equiv \hat{\mu}' \left( \frac{\iota}{\mathbf{1}'\iota} - \frac{\mathbf{1} - \iota}{m - \mathbf{1}'\iota} \right) \tag{14}$$

*where $\iota$ is an $m$-vector with $\iota_i = \chi_{[\hat{\mathbf{c}}_i > Q_{70}(\hat{\mathbf{c}})]}$ (i.e. $\iota_i = 1$, if fund $i$ is among the top 30% of funds ranked by correlation and $\iota_i = 0$ otherwise).*

If BDI measures the information not embedded in the current stock prices and this missing information transmits to prices over time, we conjecture that empirically one would observe that stocks with large positive (negative) BDI statistics have larger (smaller) future returns as prices eventually incorporate the positive (negative) information.

In the rest of the paper, we extract revealed beliefs from mutual fund stock holdings and test the above empirical implications regarding BDI.

[5] In this analysis we only consider those stocks for which we are able to obtain portfolio revealed beliefs from both sets of managers.

# 4 Data and Methods

In this section, we begin by describing the data set and the sample selection criteria. We then describe the methodology used to extract fund managers' beliefs. Finally, we describe the tests used to evaluate our predictions.

## 4.1 Sample

We employ four databases: CRSP stock daily return file, CRSP stock monthly return file, CRSP mutual fund monthly return file and the stock holdings of mutual funds from the CDA/Spectrum Mutual Fund Holding database maintained by Thomson Financial from January 1980 to December 2006. The mutual fund holding database comprises mandatory SEC filings as well as voluntary disclosures by mutual funds. It is typically available quarterly. Wermers (2000) describes this database in more detail.

For this study, we focus on domestic all-equity funds. To construct the sample, we begin with quarterly fund holdings from the CDA/Spectrum Mutual Fund Holding database.[6] We restrict our sample to only those fund-quarters where the Investment Object Code reported by CDA/Spectrum is: aggressive growth, growth, growth and income, unclassified, or missing. We remove all observations where the number of shares held is missing, where the CUSIP of the held security is missing, or where the CUSIP cannot be matched to the CRSP monthly stock return file. We also eliminate any funds that cannot be matched to a fund tracked in the CRSP monthly mutual fund file.[7]

Finally, we eliminate any fund-quarters where the fund's equity holdings amount to less than $10 million in year 2000 dollars (where the fund holdings are adjusted for inflation), or where the fund holds fewer than 20 stocks.[8] A year by year summary of the sample is found in Table 1. We also provide a comparison between the size of our stock sample and the size of the CRSP universe.

[6]Throughout, our quarters are calendar year quarters. Any reported holding date in CDA/Spectrum that does not fall on a calendar quarter end is adjusted (into the future) so that it does.

[7]Matching the CDA/Spectrum holdings to the CRSP monthly mutual fund file is done using the MFLINKS programs provided by Wermers (2000)

[8]The fund's equity holding and number of stocks held is calculated from the reported holdings in CDA/Spectrum. The calculation considers only those stocks that can be matched to the CRSP stock monthly file.

## 4.2 Extracting Beliefs

From Result 1, the extraction of fund managers' beliefs requires five elements: the manager's portfolio holdings, the variance-covariance matrix, the fund's benchmark portfolio, the fund's performance target (the expected active return), and a horizon over which to estimate expected returns. Once these elements are known, the calculation is trivial. Hence, we focus here on our handling of these information requirements.

### Portfolio Holdings

Our ability to observe portfolio holdings is a major constraint on the methodology. Specifically, we are only able to observe portfolio holdings at the frequency available in the CDA/Spectrum database. Funds typically report holdings on a quarterly basis, so in the best case, our methodology is limited to generating quarterly beliefs.[9] This will obviously limit the power of our tests, as we will be observing no more than 108 quarters to extract beliefs for each fund manager. Another issue is that some fund holdings do not correspond to domestic equity issues for which we have readily available return data. These might be foreign securities, ADRs, bonds, commercial paper, etc.. Theory would require that these securities be included in the analysis, but this is not practical. We deal with this problem by ignoring these holdings in the analysis. This can be justified by noting that our analysis focuses on U.S. equity funds. Such funds typically have negligible holdings in these types of securities.

A similar problem stems from the fact that the portfolio holdings reports do not include information about balances of cash and cash equivalents at quarterly frequencies. This is potentially troublesome: Unlike the standard portfolio optimization problem, the optimal portfolio of the benchmark-tracking fund manager is *not* a convex combination of a tangency portfolio and the risk-free asset. The consequence of ignoring cash holdings is a biased estimation of beliefs. That said, we will largely ignore this issue, since that the bias that is introduced will not be large, and it is unusual for the types of funds that we are considering to hold large cash balances. In any event, ignoring cash holdings will only bias against finding any significant predictive power in our measures.

[9] At the end of each calendar quarter, we will generate beliefs for all funds for which recent (less than one year old) holding data is available. If multiple recent holding reports are available, we use the most recent. Some funds do not report on a quarterly basis, hence the beliefs generated for these funds may be more "stale."

**Covariance Matrix**

The second challenge in our methodology is obtaining an accurate estimate of the variance-covariance matrix. With financial data, this is always a problematic proposition. The culprit is the relatively small number of historical observations ($T$) given the large number of securities ($N$) for which covariances have to be estimated. Typically, $N$ is on the order of a few thousand; while with ten years of monthly data, $T = 120$. When $T < N$, conventional variance-covariance estimation (using the sample covariances) will produce a matrix that is singular, not positive semi-definite, and whose eigenvalues bear little resemblance to the originals.[10] To address this issue we resort to a multiple factor model of the covariance matrix.

Multiple factor model have the advantage of being significantly simpler to estimate and—more importantly—are likely to be more representative of the covariance estimators in use by mutual fund industry.[11] Since our goal is to extract the beliefs of a mutual fund manager by reverse-engineering his optimization problem, our goal is not to develop the "best" covariance matrix estimator. Rather, we are interested in using a covariance estimator that is as similar as possible to the one used by the mutual fund manager in question.

We model the covariance structure in stock returns using 53 factors. These include the three Fama French (1993) factors, excess market return, MKTRF, small-minus-big, SMB, and high-minus-low (book-to-market), HML. the momentum factor. We also include the momentum, or up-minus-down factor, UMD of Carhart (1997). Finally, we include the returns on the 49 industry portfolios available from Ken French's website. Thus, the data generating process for excess returns is taken to be

$$\tilde{\mathbf{r}} = \alpha + \beta\tilde{\mathbf{f}} + \tilde{\mathbf{e}}$$

where $\tilde{\mathbf{f}}$ is the return to the factors, with covariance matrix $\Sigma_f$, and $\tilde{\mathbf{e}}$ is the vector of idiosyncratic returns with zero mean and a diagonal variance structure ($Var[\tilde{\mathbf{e}}] = diag(\sigma_1^2, \ldots, \sigma_n^2)$). The matrix $\beta$ is taken to be the factor loadings on each of the 53 factors for each of the securities. Idiosyncratic returns are assumed to be uncorrelated with the factor returns,

[10] Schäfer and Strimmer (2005) provide some simulation results that demonstrate the severity of the problem.

[11] Risk models sold to the mutual fund industry by vendors like MSCI Barra typically feature multi-factor covariance matrix estimators.

hence the total variance of returns is:

$$\Sigma = \beta \mathbf{\Sigma_f} \beta' + diag(\sigma_1^2, \ldots, \sigma_n^2). \tag{15}$$

To estimate the factor covariance matrix, $\Sigma_f$ we use the sample covariance estimator. To estimate the factor loadings, $\beta$, we regress each security's returns on the contemporanous returns of the 53 factors. We then use the coefficient estimates from these regressions as estimates of the factor loadings. To estimate $\sigma_1^2, \ldots, \sigma_n^2$ we use the sample variances of the residuals from each of the factor-loading regressions.

It would be optimistic to suppose that the variance-covariance matrix is stationary for extended periods. In our model this manifests itself via changes to the covariance of the factor returns, changes to the factor loadings for each security, and changes to the idiosyncratic volatility of each security. To address this, we use relatively short, five year windows to estimate all covariances and factor loadings. In order to maintain a sufficient number of observations we resort to higher frequency return data. Doing so introduces the problem of non-synchronous trading effects. Lo and MacKinlay (1990) show that small stocks may not react to common market news for days, or even weeks. Covariance estimates that do not take this into account understate the degree of co-movement. To deal with this issue, we follow the convention and calculate weekly (Wednesday to Wednesday) returns from the CRSP daily return file. This increases the number of observations four-fold without incurring the brunt of the the non-synchronous trading bias.

In our analysis, we generate a new variance-covariance estimate, $\hat{\mathbf{\Sigma}}$, for each quarter based on the previous five years of weekly returns. For missing weekly return observations, we assume the risk-free rate.

### Benchmark Portfolio

Up to this point we have considered the benchmark portfolio, $\mathbf{q}_m$, as given. Unfortunately, empirical realities are quite a bit different. There is no reliable source for fund benchmarks over the entire period under consideration. Furthermore where such data are available there is no guarantee that they are accurate. Funds may (for various reasons) say one thing, and do quite another. Given this, our approach is to let the holdings data speak: If the fund holds or has recently (in the last five years) held some security, then that security is considered to belong to the benchmark. We set the benchmark weights based on market capitalization, thus the fund's benchmark is a value-weighed index of securities in which the fund has shown any interest in the last five years.

Our benchmark selection methodology will include all securities that are part of the true benchmark. This follows from the premise that the manager is mean-variance optimizing. If that is the case, then the solution to the optimization problem will inevitably suggest some non-zero position for every security. Although one could argue that the manager will not hold certain negative positions due to short-sale constraints, in our setting this is not a major issue. Unlike a mean-variance investor, our fund manager will rarely run up against a short-sale constraint. Loosely speaking, this is because to our fund managers, any underweighing of a security relative to his benchmark is effectively a short position.[12]

### Performance Target

A fund manager's performance target is required to fully back out his beliefs. Without this, we are only able to obtain his beliefs up to a multiplicative constant. If we only evaluate how closely a fund manager's belief correlates with ex post stock returns, this is not an issue as the correlation is unaffected by scale. However, if we compare *differences* in beliefs across fund managers, proper scaling of the beliefs is important. In our analysis we deal with this by assuming that while fund managers may have different beliefs on the returns of individual securities, they have common beliefs on the dispersion of returns—the standard deviation of beliefs is assumed constant across managers. Thus, scaling each manager's revealed beliefs, $\hat{\mu}_m$ by the standard deviation of his beliefs $((\hat{\mu}_m - \mathbf{1}'\hat{\mu}_m/n)'(\hat{\mu}_m - \mathbf{1}'\hat{\mu}_m/n)/(n-1))$ obtains a measure of belief that is comparable across all managers. We refer to these as *normalized beliefs*.[13]

### Horizon

Given that we observe funds' holdings data quarterly, we update our estimates of *revealed belief* using the holding data at the end of each quarter. Assuming that a quarter ends

[12] Note that Result 1 indicates that managers have no revealed beliefs on stocks that are not in their benchmarks. That is. if a manager does not hold a stock *and* this stock is not in his benchmark, then we regard this manager does not have a view on this stock. Of course, a zero holding on a stock which is in his benchmark would reveal his negative belief (risk-adjusted) of this stock's expected excess return by Result 1.

[13] Essentially this normalization assumes that managers agree about the dispersion of aggregation stock returns. This is similar to the assumption made by Black and Litterman (1992) where they assume that investors agree that market portfolio reflects equilibrium beliefs of expected returns.

at the end of month $t$, we take the funds' reported holdings at quarter end and calculate $\hat{\mu}_m$ from those data. To rank fund managers' stock picking ability, we correlate realized returns in month $t+1$ with the fund's $\hat{\mu}_m$ for all the stocks in the fund's holding. We rank managers based on the average of such correlations for past few quarters (The reason that we use correlations for more than one quarter is to make sure that we rank funds according to their relatively persistent stock picking abilities). To compute a stock's BDI, we take the difference between the mean of top-30% ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock.[14] Finally, we evaluate the forecasting ability of BDI over months $t+2$, $t+3$ and $t+4$. Some of the forecasting ability of BDI appears to persist for a few months after $t+4$, but it weakens with forecast horizon.

Specifically, we report the results for the following exercises in the paper. At the end of each calendar quarter (end of March, June, September, and December), we calculate the beliefs for each fund manager in the sample using the latest available holding data. We compute the correlation measure by correlating these beliefs with the ex-post returns in the subsequent month (end of April, July, October, and January).[15] We then rank funds into deciles by the average of such correlations for the last four quarters, compute BDI and sort stocks into deciles by BDI. We form equal- and value-weighted decile portfolios based on BDI and evaluate their performance over the next three months (May to July, August to October, November to January, and February to April).

# 5 Results

The primary aim of our empirical analysis is to determine the extent to which the information in funds' holdings can be used to predict stock returns. In this section, we test the predictability of BDI for future stock returns and present the findings.

[14]We find that funds in the extremely-ranked deciles have smaller sizes than the rest. For example, for the subsample of 2000-2005, the average sizes of funds from decile 1 to 10 are: 781229, 10001.563, 1160.779, 1125.531, 1050.595, 1002.403, 1100.510, 1178.919, 1004.754, 817.052 million dollars, respectively. The subsamples of 1980-1990 and 1990-2000 have similar distributions.

[15]Note that we correlate one-month realized returns with expected returns. This one month horizon is similarly used in Spiegel, Mamaysky, and Zhang (2007) for their back testing method.

## 5.1 Revealed Beliefs and Future Stock Returns

To test the predictability of BDI we sort stocks into deciles based on BDI each quarter. From these deciles, we construct equal-weight and value-weight portfolios and evaluate the performance for three months starting with the second month of the quarter.

The performance measures we consider include excess returns, alphas from the one factor CAPM model of Sharpe (1963), three-factor model of Fama and French (1993), and four-factor model of Carhart (1997), as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). For the various measures of alpha, we run standard time-series regressions. We use the Newey-West standard errors to deal with possible auto-correlation in the error terms.

The risk- and style-adjusted net returns for each equal-weight decile portfolio are reported in Table 2. The second column in this table reports the average returns for stocks in each BDI decile. The next column reports the excess returns (that is, returns over the risk-free rate). The next three columns report the intercepts from a time-series regression based on the one-factor CAPM model, the three-factor model of Fama and French (1993), the four-factor model of Carhart (1997), and the characteristic selectivity measure of Daniel, Grinblatt, Titman, and Wermer (1997), respectively.

Table 2 shows that future stock performance for each BDI decile. These results show that the difference in beliefs on future returns between the informed managers and the less informed managers is a good predictor of future stock returns: Stocks in the top decile outperform those in the bottom decile. Investing in stocks in the top decile and shorting those in the bottom decile results in a significant monthly return of between 23 and 36 basis points or 2.76 and 4.32 percent per annum, depending on the measure. These results are all statistically significant at the five percent level, and most are significant at the one percent level. They do not seem to be explained by risk factor loadings. Strongest results obtain for the difference between top decile stocks and bottom decile stocks. Results for the difference between the nine-to-ten and one-to-two deciles are somewhat weaker— 7 to 19 basis points per month—but still generally significant at the five percent level (except for the Carhart alpha).

Further, much of the return predictability in BDI is driven by stocks in the highest BDI deciles. Looking specifically at the returns reported in Table 2, it is clear that most of the positive alphas reported for BDI decile 10 of stocks are significantly positive. This is important, since it is sometimes costly to short stocks. Apparently a significant portion of the return predictability that we document is driven by successful fund managers that choose stocks that outperform the market.

Table 3 shows the characteristics of the equal-weighted BDI decile portfolios. The loading on the market factor is very close to one for almost all the deciles. The worst performing decile has the highest market beta. The factor loadings for SMB follow a "U" shape and for HML and UMD an inverted "U" shape. That is, stocks in the middle deciles—those stocks on which the informed and less informed managers agree—are bigger, have higher book-to-market, and have performed better in the past than the stocks on which there is no consensus.[16] In other words, there appears to be more disagreement about small, growth stocks, with poor performance track records. This is consistent with intuition.

Table 4 reports the performance results for the value-weight decile portfolios. The difference between the top and bottom value-weight deciles is a bit larger than the difference between the top and bottom equal-weight deciles in magnitude and statistical significance for all measures. The results for the difference between the top and bottom deciles are between 38 and 45 basis points per month, approximately ten basis points higher than the equal-weight results, or 4.56 and 5.4 percent per annum. This is somewhat unexpected as value-weighing tends to emphasize the large stocks for which it should be comparatively more difficult to obtain information not already known to the market. We investigate this issue further in Table 6.

The factor loadings for the value-weight deciles reported in Table 5 are similar to the equal-weight results, except for the momentum factor. Here, momentum loadings depart from the "U" shape. The top decile portfolio has a positive momentum factor loading while the bottom portfolio has a negative loading. Informed managers appear to prefer large stocks that have performed well recently to those that have performed poorly.

To investigate why value-weighted portfolios yield better performance than equal-weighted portfolios, we sort stocks into three groups: small, medium, and big according to their size at the end of each quarter and redo our analysis in Table 4. That is, for each size group, we sort stocks into deciles based on BDI. From these deciles, we construct value-weight portfolios and evaluate the performance for three months starting with the second month of the quarter. The findings are reported in Table 6. The results show that the significant

[16]More precisely, the stocks in the "consensus" deciles have a lower loading on the size factor, and higher loadings on the value and momentum factors.

performance difference between the top and bottom deciles reported in Table 4 comes from the medium size group. In fact, the performance difference between the top and the bottom decile is marginally *negative* for the small stocks, and insignificant for the large stocks. Since value-weighted portfolios under-weight the under-performance of small stocks, these performance differences explain why the value-weighted portfolios yield a better performance (Table 4) than the equal-weight portfolios (Table 2). This finding indicates that the reverse-engineering approach does not work well for very small or very large stocks. Intuitively, when incorporating small stocks into their portfolios, fund managers may not follow the risk-return optimization since these stocks typically consist of a small fraction of the fund portfolio. The findings in Table 6 about the large stocks are also consistent with our prior: fund managers do not appear to have large disagreement about large stocks. These results suggest that the stock picking skills of fund managers are reflected mostly in medium size stocks.

### 5.2 Robustness Checks: Alternative Estimates of Variance-Covairnace Matrix, Return Gap and Short-term Continuation

Overall the evidence supports our conjecture that BDI contains information that is valuable for forecasting stock returns. We also conduct various robustness checks about our results. First, we use several alternative methods to estimate the variance-covariance matrix. For example, we estimate revealed beliefs using a diagonal variance-covariance matrix where the diagonal terms are estimated using sample variances of the corresponding stock returns. The results are weaker when we ignore the off-diagonal terms, as shown in Table 7. We also estimate the revealed beliefs using a "shrinkage" estimator of the variance-covariance matrix as in Ledoit and Wolf (2003). The shrinkage estimator yields bigger alphas, but at a lower level of statistical significance.[17]

We also estimate revealed beliefs by using the identity matrix as the variance-covariance matrix. By doing so, we are effectively assuming that fund managers do not adjust for risks when constructing their portfolios and hence the portfolio weights reflect their revealed beliefs on the expected future returns of corresponding stocks. The results are significantly weaker as shown in Table 8: The performance differentials between the top and the bottom deciles are about half as before and the Carhart alphas are no longer significant. In fact, the

[17]Results are available upon requests.

Carhart alpha difference between the nine-to-ten and one-to-two deciles becomes negative. Note that by ignoring the risk aspect of portfolio optimization, this approach adopts logic similar to that of Grinblatt and Titman (1989) where the correlation of the change in portfolio holdings with subsequent realized returns serves as a proxy for managerial abilities—the GT measure. They find that the GT measure is able to forecast future fund performance. Our study shows that by ignoring the fact that the fund managers adjust for risk in choosing optimal portfolios, the forecasting ability of BDI for future stock returns is limited.[18]

Since our paper studies the information content of revealed beliefs embedded in portfolio holdings, it would be interesting to see whether our results hold for funds whose returns computed from reported quarterly holdings are not too far off from realized returns, i.e., the funds with smaller return gaps (Kacperczyk, Sialm, and Zheng (2008)). To do so, we exclude funds which are ranked top 20% in terms of return gaps (in absolute terms) and rerun our analysis. The equal-weight portfolio results are reported in Table 9.

Finally, we also ensure that the results are not driven by short-term return continuations. Given the nature of our tests, it is possible that the results are driven by some sort of short-term return continuation rather than fund manager stock picking skills. Suppose, for example, that the managers, whose revealed beliefs are highly correlated with the subsequent one-month realized returns, have no particular skill, but are randomly lucky in any given quarter to have a high correlation between their revealed beliefs and returns in the subsequent month. If the stocks that have done well in one month continue to have relatively high returns in the subsequent quarter, a positive correlation between BDI and subsequent stock returns may result. While this story appears to go against the literature on short-term return reversals, it is easy to check whether it drives the results.

We check the robustness of the results by delaying all the holding reports by two quarters and running the entire analysis again. That is, we treat holdings reported at quarter $t$ as if they are reported at quarter $t+2$. We then construct BDI using these lagged portfolios. According to our model, the information embedded in these lagged portfolios is stale and should not reflect managerial skills. Consequently, none of results should obtain. Alternatively, if our results are driven by the return continuation story described above, BDI constructed using the these "random" portfolios could be related to future stock return. Our analysis confirms that the former is the case. We find that using lagged portfolio holdings to perform our test reduces the return predictability of our tests to close to zero. This gives

[18] Similar comparison also applies to Chen, Jegadeesh, and Wermers (2000).

us confidence that the predictability we document truly results from manager ability rather than from the nature of our model and estimation algorithm. For brevity, we do not report the numerical results.

## 6 Conclusion

This paper aims to examine the information content of revealed beliefs of mutual fund managers. The revealed beliefs are backed out by reverse-engineering fund manger's portfolio optimization problem. Specifically, to elicit the revealed beliefs, we assume that each manager rationally optimizes over the risk return tradeoff relative to his own benchmark portfolio. The key idea is that managers tilt their portfolios toward stocks with better risk-return tradeoffs according to their private beliefs. Hence, observing their holdings, one can determine whether fund managers' beliefs on future returns are accurate.

Based on these revealed beliefs, we propose to measure the differences in beliefs between minority informed funds (those with higher correlated revealed beliefs with subsequent realized returns) and the rest, which is BDI. The evidence in this paper suggests that investors could profit from extracting fund manager information about individual stock returns through the BDI measure. Theoretically, the BDI measure contributes to the general finance literature by demonstrating it is possible to extract information contained in cross-sectional fund holdings through exploiting a portfolio optimization framework. Empirically, the result on the predictability of BDI over future stock returns suggest that—to a certain degree—professional money managers do adjust for risk in their portfolio allocation decisions, hence indicating the importance of modern portfolio theory in practice.

More fundamentally, the paper makes a unique contribution to the finance literature by introducing a revealed preference approach to measuring investor expectations. That is, instead of estimating investor expectation regarding risk and returns from historical returns, we show that it can be useful to back out investors' expectations regarding returns (and potentially risks) from their portfolio holdings. This revealed belief is forward-looking, inherently dynamic, and heterogenous among investors. This approach may be of great empirical importance for future work. For example, various strands of the market microstructure literature are built on the assumption that investors are heterogeneously or asymmetrically informed. Without a concrete measure of investor beliefs, most empirical tests of these theories are based on equilibrium price patterns, which might suffer from endogeneity and measurement error. Having a relatively direct measure of investor beliefs might help researchers identify the degree of information asymmetry at a point in time or among a set of investors. Similarly, the asset pricing literature often involves estimation of dynamically changing expected returns. The information provided in portfolio holdings on investor belief about expected future returns might improve existing estimation techniques, and hence have important implications for empirical asset pricing.

## References


Berk, Jonathan, and Richard Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.

Black, Fisher, and Bob Litterman, 1992, Global portfolio optimization, *Financial Analysts Journal* 48, 28–43.

Breon-Drish, Bradyn, and Jacob Sagi, 2008, Do fund managers make informed asset allocation decisions?, Vanderbuilt University Working Paper.

Carhart, Mark M., 1997, On persistence in mutual fund performance, *Journal of Finance* 50, 679–698.

Chen, Hsiu-Lang, Narasimhan Jegadeesh, and Russ Wermers, 2000, The value of active mutual fund management: An examination of the stockholdings and trades of fund managers, *Journal of Financial and Quantitative Analysis* 35, 343–368.

Cohen, Randy, Christopher Polk, and Bernhard Silli, 2009, Best Idea, London School of Economics working paper.

Cohen, Randy B., Joshua D. Coval, and L. Pastor, 2005, Judging fund managers by the company they keep, *Journal of Finance* 60, 1057–1096.

Cremers, Martijn, and Antti Petajisto, 2006, How active is your fund manager? A new measure that predicts performance, *The Review of Financial Studies* forthcoming.

Daniel, Kent, M. Grinblatt, Sheridan Titman, and Russ Wermers, 1997, Measuring mutual fund performance with characteristic-based benchmarks, *Journal of Finance* 52, 1035–1058.

Fama, Eugene, and Kenneth French, 1993, Common risk factors in returns on stocks and bonds, *Journal of Financial Economics* 51, 3–56.

Ferson, Wayne E., and Kenneth Khang, 2002, Conditional performance measurement using portfolio weights: Evidence for pension funds, *Journal of Financial Economics* 65, 249–282.

Goetzmann, William N., and Massimo Massa, 2006, Behavior factors in mutual fund flows, Yale International Center for Finance Working Paper.

Graham, John, and Campbell Harvey, 1996, Market timing ability and volatility implied in investment newsletters' asset allocation recommendations, *Journal of Financial Economics* 42, 397–421.

Grinblatt, M., and Sheridan Titman, 1989, Mutual fund performance: An analysis of quarterly portfolio holdings, *Journal of Business* 62, 393–416.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the Impossibility of Informationally Efficient Markets, *The American Economic Review* 70, 393–408 ArticleType: primary_article / Full publication date: Jun., 1980 / Copyright 1980 American Economic Association.

Kacperczyk, Marcin, and Amit Seru, 2007, Fund manager use of public information: New evidence on managerial skills, *Journal of Finance* 62, 485–528.

Kacperczyk, Marcin T., Clemens Sialm, and Lu Zheng, 2005, On industry concentration of actively managed equity mutual funds, *Journal of Finance* 60, 1983–2011.

Kacperczyk, Marcin T., Clemens Sialm, and Lu Zheng, 2008, Unobserved actions of mutual funds, *Review of Financial Studies* 21, 2379–2416.

Ledoit, Olivier, and Michael Wolf, 2003, Improved estimation of the covariance matrix of stock returns with an application to portfolio selection, *Journal of Empirical Finance* 10, 603–621.

Lo, Andrew, and Jiang Wang, 2000, Trading volume: Definition, data analysis, and implications of portfolio theory, *Review of Financial Studies* 13, 257–300.

Lo, Andrew, and Jiang Wang, 2001, Trading volume: Implications of an intertemporal capital asset pricing model, MIT Sloan School of Management Working Paper.

Lo, Andrew W., and A. Craig MacKinlay, 1990, When are contrarian profits due to stock market overreaction?, *The Review of Financial Studies* 3, 175–205.

Markowitz, Harry, 1952, Portfolio selection, *Journal of Finance* 7, 77–91.

Roll, R., 1992, A mean/variance analysis of tracking error, *Journal of Portfolio Management* 18, 13–22.

Schäfer, J., and K. Strimmer, 2005, A shrinkage approach to large-scale covariance matrix estimation and implications for functional genomics, *Statistical Applications in Genetics and Molecular Biology* 4, 32.

Sharpe, William F., 1963, A simplified model for portfolio analysis, *Management Science* 9, 277–293.

Spiegel, Matthew, Harry Mamaysky, and Hong Zhang, 2007, Improved forecasting of mutual fund alphas and betas, *Review of Finance* 11, 359–400.

Wermers, Russ, 2000, Mutual fund performance: An empirical decomposition into stock picking talent, style, transaction costs, and expenses, *Journal of Finance* 55, 1655–1695.

Wermers, Russ, Tong Yao, and Ji Zhao, 2007, The investment value of mutual fund portfolios, University of Maryland Working Paper.

Table 1: Summary Statistics: Sample

For each year (row) below, descriptive statistics are presented on the funds in the sample. We begin with mutual fund holdings data provided by CDA/Thomson Financial. We eliminate non-equity funds (based on IOC code), any funds that do not appear in the CRSP mutual fund monthly file, funds that hold fewer than 20 stocks, and funds that have equity holdings totaling less than $10 million (in year 2000 dollars) under management. We report the number of distinct funds and the average number of stocks held by a fund. We also report the number of distinct stocks and the market values of stocks in our sample and in the CRSP universe and the proportion of the CRSP universe covered by our sample.

| | Mutual Fund Sample | | | | CRSP Universe | | Proportion | |
|---|---|---|---|---|---|---|---|---|
| Year | Number of Funds | Number of Distinct Stocks | Average Number of Stocks | Market Value of Stocks Held ($B) | Number of Distinct Stocks | Total Market Capitalization ($B) | Number of Distinct Stocks (%) | Value of Stocks (%) |
| 1980 | 260 | 2180 | 57 | 37.22 | 5242 | 1522.78 | 41.59 | 2.44 |
| 1981 | 254 | 2295 | 61 | 33.02 | 5600 | 1422.76 | 40.98 | 2.32 |
| 1982 | 235 | 2508 | 60 | 39.28 | 5732 | 1644.56 | 43.75 | 2.39 |
| 1983 | 261 | 3186 | 70 | 55.29 | 6399 | 1898.73 | 49.79 | 2.91 |
| 1984 | 291 | 3306 | 71 | 59.9 | 6513 | 1801.77 | 50.76 | 3.32 |
| 1985 | 319 | 3572 | 72 | 84.61 | 6550 | 2243.48 | 54.53 | 3.77 |
| 1986 | 370 | 3832 | 75 | 102.94 | 6964 | 2515.87 | 55.03 | 4.09 |
| 1987 | 414 | 3858 | 77 | 112.25 | 7350 | 2517.91 | 52.49 | 4.46 |
| 1988 | 436 | 4103 | 80 | 122.04 | 7197 | 2752.81 | 57.01 | 4.43 |
| 1989 | 460 | 4066 | 81 | 131.45 | 6998 | 3356.61 | 58.1 | 3.92 |
| 1990 | 476 | 3753 | 83 | 129.26 | 6849 | 3032.87 | 54.8 | 4.26 |
| 1991 | 582 | 3998 | 88 | 222.3 | 7000 | 4071.97 | 57.11 | 5.46 |
| 1992 | 686 | 4221 | 98 | 294.3 | 7155 | 4476.72 | 58.99 | 6.57 |
| 1993 | 848 | 5614 | 111 | 402.7 | 7978 | 5184.07 | 70.37 | 7.77 |
| 1994 | 1051 | 6159 | 121 | 472.65 | 8397 | 5123.17 | 73.35 | 9.23 |
| 1995 | 1087 | 6571 | 125 | 706.22 | 8667 | 6927.22 | 75.82 | 10.19 |
| 1996 | 1317 | 7148 | 119 | 1021.7 | 9248 | 8498.34 | 77.29 | 12.02 |
| 1997 | 1556 | 7540 | 121 | 1404.74 | 9342 | 11054.25 | 80.71 | 12.71 |
| 1998 | 1497 | 7361 | 121 | 1745.52 | 8954 | 13566.81 | 82.21 | 12.87 |
| 1999 | 1815 | 6619 | 119 | 2347.19 | 8608 | 17603.64 | 76.89 | 13.33 |
| 2000 | 2114 | 7255 | 131 | 2587.23 | 8372 | 16067.67 | 86.66 | 16.1 |
| 2001 | 1642 | 6332 | 136 | 1526.92 | 7647 | 14211.4 | 82.8 | 10.74 |
| 2002 | 1847 | 5899 | 139 | 1347.19 | 7243 | 11311 | 81.44 | 11.91 |
| 2003 | 2115 | 5879 | 143 | 2220.66 | 6899 | 14998.79 | 85.22 | 14.81 |
| 2004 | 2035 | 5792 | 146 | 2390.94 | 6895 | 16957.1 | 84 | 14.1 |
| 2005 | 1982 | 5744 | 148 | 2567.23 | 6888 | 18004.8 | 83.39 | 14.26 |
| 2006 | 1954 | 5863 | 146 | 3118.51 | 6984 | 20388.93 | 83.95 | 15.3 |

Table 2: Equal-Weight BDI-Decile Stock Portfolios: Performance

One month into each quarter we sort stocks into deciles based on the difference between the mean of top-30% correlation ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock using the previous four quarters' holding reports. Based on these (BDI) deciles we create equal-weight portfolios that we buy at the start of the second month of the quarter. We hold the portfolio for 3 months, repeating the procedure in the subsequent quarter. For each decile portfolio, we present the monthly average returns, the excess returns (over the risk free rate), the CAPM, Fama-French, and Carhart alphas, as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). We also provide results for long-short portfolios constructed by buying the top one (two, five) decile(s) and selling the bottom one (two, five) decile(s). Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | Average Return | Excess Return | CAPM Alpha | Fama-French Alpha | Carhart Alpha | DGTW CS |
|---|---|---|---|---|---|---|
| 1 | 0.0111 | 0.0064 | -0.0019 | -0.0031 | 0.0009 | -0.0002 |
| | (0.0039)** | (0.0039)† | (0.0021) | (0.0014)* | (0.0016) | (0.0012) |
| 2 | 0.0129 | 0.0082 | 0.0007 | -0.0011 | 0.0016 | 0.0005 |
| | (0.0033)** | (0.0033)** | (0.0016) | (0.0011) | (0.0010)† | (0.0007) |
| 3 | 0.0130 | 0.0083 | 0.0012 | -0.0007 | 0.0012 | 0.0008 |
| | (0.0030)** | (0.0030)** | (0.0015) | (0.0010) | (0.0008)† | (0.0007) |
| 4 | 0.0138 | 0.0091 | 0.0020 | 0.0001 | 0.0016 | 0.0016 |
| | (0.0030)** | (0.0030)** | (0.0015)† | (0.0008) | (0.0008)* | (0.0006)** |
| 5 | 0.0139 | 0.0092 | 0.0024 | 0.0001 | 0.0013 | 0.0014 |
| | (0.0029)** | (0.0029)** | (0.0015)† | (0.0008) | (0.0008)† | (0.0006)** |
| 6 | 0.0129 | 0.0082 | 0.0014 | -0.0012 | -0.0000 | 0.0002 |
| | (0.0029)** | (0.0029)** | (0.0016) | (0.0009)† | (0.0008) | (0.0006) |
| 7 | 0.0138 | 0.0091 | 0.06022 | -0.0002 | 0.0012 | 0.0011 |
| | (0.0030)** | (0.0030)** | (0.0016)† | (0.0008) | (0.0008)† | (0.0005) * |
| 8 | 0.0128 | 0.0081 | 0.0010 | -0.0014 | 0.0001 | 0.0001 |
| | (0.0031)** | (0.0031)** | (0.0016) | (0.0007)* | (0.0008) | (0.0005) |
| 9 | 0.0128 | 0.0081 | 0.0008 | -0.0009 | 0.0007 | 0.0006 |
| | (0.0032)** | (0.0032)** | (0.0016) | (0.0007) | (0.0008) | (0.0006) |
| 10 | 0.0141 | 0.0094 | 0.0012 | 0.0005 | 0.0032 | 0.0027 |
| | (0.0039)** | (0.0039)** | (0.0019) | (0.0011) | (0.0013)** | (0.0010)** |
| Top 10% - | 0.0030 | 0.0030 | 0.0031 | 0.0036 | 0.0023 | 0.0030 |
| Bottom 10% | (0.0009)** | (0.0009)** | (0.0009)** | (0.0010)** | (0.0010)* | (0.0010)** |
| Top 20% - | 0.0014 | 0.0014 | 0.0016 | 0.0019 | 0.0007 | 0.0015 |
| Bottom 20% | (0.0007)* | (0.0007)* | (0.0007)** | (0.0008)* | (0.0008) | (0.0007) * |

Table 3: Equal-Weight BDI-Decile Stock Portfolios: Four Factor Loadings

For each decile portfolio in Table 2, we present the intercept and the "betas" for market, small-minus-big (SMB), high book-to-market minus low book-to-market (HML), and momentum (UMD) portfolios. Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | N | Alpha | Market | SMB | HML | UMD |
|---|---|---|---|---|---|---|
| 1 | 320 | 0.0009 | 1.1079 | 0.9298 | 0.0080 | -0.3987 |
| | | (0.0016) | (0.0370)** | (0.0628)** | (0.0779) | (0.0765)** |
| 2 | 320 | 0.0016 | 1.0895 | 0.7031 | 0.1464 | -0.2679 |
| | | (0.0010)† | (0.0260)** | (0.0535)** | (0.0484)** | (0.0407)** |
| 3 | 320 | 0.0012 | 1.0582 | 0.6009 | 0.1910 | -0.1956 |
| | | (0.0008)† | (0.0210)** | (0.0542)** | (0.0505)** | (0.0333)** |
| 4 | 320 | 0.0016 | 1.0667 | 0.5582 | 0.2110 | -0.1505 |
| | | (0.0008)* | (0.0196)** | (0.0625)** | (0.0506)** | (0.0334)** |
| 5 | 320 | 0.0013 | 1.0718 | 0.4725 | 0.2750 | -0.1180 |
| | | (0.0008)† | (0.0258)** | (0.0781)** | (0.0596)** | (0.0378)** |
| 6 | 320 | -0.0000 | 1.0707 | 0.5073 | 0.3126 | -0.1114 |
| | | (0.0008) | (0.0257)** | (0.0728)** | (0.0624)** | (0.0389)** |
| 7 | 320 | 0.0012 | 1.0666 | 0.5839 | 0.2653 | -0.1391 |
| | | (0.0008)† | (0.0199)** | (0.0522)** | (0.0417)** | (0.0310)** |
| 8 | 320 | 0.0001 | 1.0864 | 0.6190 | 0.2703 | -0.1507 |
| | | (0.0008) | (0.0231)** | (0.0553)** | (0.0484)** | (0.0316)** |
| 9 | 320 | 0.0007 | 1.0560 | 0.7327 | 0.1614 | -0.1612 |
| | | (0.0008) | (0.0162)** | (0.0457)** | (0.0402)** | (0.0336)** |
| 10 | 320 | 0.0032 | 1.0822 | 0.9324 | -0.0297 | -0.2757 |
| | | (0.0013)** | (0.0264)** | (0.0451)** | (0.0533) | (0.0559)** |
| Top 10% - | 320 | 0.0023 | -0.0258 | 0.0025 | -0.0378 | 0.1230 |
| Bottom 10% | | (0.0010)* | (0.0267) | (0.0429) | (0.0512) | (0.0358)** |
| Top 20% - | 320 | 0.0007 | -0.0296 | 0.0160 | -0.0115 | 0.1149 |
| Bottom 20% | | (0.0008) | (0.0206)† | (0.0302) | (0.0372) | (0.0266)** |

Table 4: Value-Weight BDI-Decile Stock Portfolios: Performance

One month into each quarter we sort stocks into deciles based on the difference between the mean of top-30% correlation ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock using the previous four quarters' holding reports. Based on these (BDI) deciles we create value-weight portfolios that we buy at the start of the second month of the quarter. We hold the portfolio for 3 months, repeating the procedure in the subsequent quarter. For each decile, we present the monthly average returns, the excess returns (over the risk free rate), the CAPM, Fama-French, and Carhart alphas, as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). We also provide results for long-short portfolios constructed by buying the top one (two, five) decile(s) and selling the bottom one (two, five) decile(s). Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | Average Return | Excess Return | CAPM Alpha | Fama-French Alpha | Carhart Alpha | DGTW CS |
|---|---|---|---|---|---|---|
| 1 | 0.0080 | 0.0033 | -0.0046 | -0.0036 | -0.0031 | -0.0021 |
| | (0.0034)* | (0.0034) | (0.0018)** | (0.0018)* | (0.0019)* | (0.0013) † |
| 2 | 0.0115 | 0.0068 | -0.0004 | 0.0004 | 0.0014 | 0.0004 |
| | (0.0029)** | (0.0029)** | (0.0010) | (0.0010) | (0.0010)† | (0.0006) |
| 3 | 0.0105 | 0.0058 | -0.0008 | -0.0012 | -0.0009 | -0.0013 |
| | (0.0025)** | (0.0025)** | (0.0009) | (0.0009)† | (0.0009) | (0.0007) * |
| 4 | 0.0118 | 0.0071 | 0.0006 | 0.0002 | -0.0001 | 0.0004 |
| | (0.0024)** | (0.0024)** | (0.0008) | (0.0008) | (0.0007) | (0.0005) |
| 5 | 0.0133 | 0.0086 | 0.0021 | 0.0017 | 0.0018 | 0.0009 |
| | (0.0024)** | (0.0024)** | (0.0008)** | (0.0008)* | (0.0009)* | (0.0005) * |
| 6 | 0.0113 | 0.0065 | 0.0001 | -0.0000 | -0.0001 | -0.0001 |
| | (0.0026)** | (0.0026)** | (0.0008) | (0.0008) | (0.0008) | (0.0006) |
| 7 | 0.0114 | 0.0067 | 0.0004 | 0.0002 | -0.0001 | 0.0000 |
| | (0.0025)** | (0.0025)** | (0.0007) | (0.0007) | (0.0007) | (0.0006) |
| 8 | 0.0117 | 0.0070 | 0.0005 | 0.0007 | 0.0009 | -0.0004 |
| | (0.0024)** | (0.0024)** | (0.0007) | (0.0008) | (0.0008) | (0.0006) |
| 9 | 0.0115 | 0.0068 | -0.0003 | -0.0001 | -0.0008 | 0.0001 |
| | (0.0028)** | (0.0028)** | (0.0010) | (0.0012) | (0.0011) | (0.0008) |
| 10 | 0.0119 | 0.0072 | -0.0006 | 0.0009 | 0.0007 | 0.0019 |
| | (0.0033)** | (0.0033)* | (0.0013) | (0.0013) | (0.0013) | (0.0012) † |
| Top 10% - | 0.0040 | 0.0040 | 0.0040 | 0.0045 | 0.0038 | 0.0040 |
| Bottom 10% | (0.0017)* | (0.0017)* | (0.0019)* | (0.0021)* | (0.0022)* | (0.0017)** |
| Top 20% - | 0.0020 | 0.0020 | 0.0021 | 0.0020 | 0.0008 | 0.0018 |
| Bottom 20% | (0.0014)† | (0.0014)† | (0.0015)† | (0.0017) | (0.0016) | (0.0012) † |

Table 5: Value-Weight BDI-Decile Stock Portfolios: Four Factor Loadings

For each decile portfolio in Table 4, we present the intercept and the "betas" for market, small-minus-big (SMB), high book-to-market minus low book-to-market (HML), and momentum (UMD) portfolios. Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | N | Alpha | Market | SMB | HML | UMD |
|---|---|---|---|---|---|---|
| 1 | 320 | -0.0031 | 1.0661 | 0.3917 | -0.1838 | -0.0452 |
| | | (0.0019)* | (0.0688)** | (0.0473)** | (0.0782)** | (0.0466) |
| 2 | 320 | 0.0014 | 1.0123 | 0.1182 | -0.1434 | -0.1032 |
| | | (0.0010)† | (0.0310)** | (0.0432)** | (0.0780)* | (0.0419)** |
| 3 | 320 | -0.0009 | 1.0201 | -0.0194 | 0.0571 | -0.0296 |
| | | (0.0009) | (0.0279)** | (0.0284) | (0.0576) | (0.0319) |
| 4 | 320 | -0.0001 | 1.0412 | -0.1423 | 0.0805 | 0.0224 |
| | | (0.0007) | (0.0215)** | (0.0480)** | (0.0503)† | (0.0253) |
| 5 | 320 | 0.0018 | 1.0232 | -0.0946 | 0.0598 | -0.0122 |
| | | (0.0009)* | (0.0289)** | (0.0439)* | (0.0440)† | (0.0253) |
| 6 | 320 | -0.0001 | 1.0189 | -0.1595 | 0.0237 | 0.0143 |
| | | (0.0008) | (0.0202)** | (0.0328)** | (0.0406) | (0.0267) |
| 7 | 320 | -0.0001 | 0.9800 | -0.0647 | 0.0457 | 0.0272 |
| | | (0.0007) | (0.0224)** | (0.0397)† | (0.0456) | (0.0260) |
| 8 | 320 | 0.0009 | 0.9788 | -0.0888 | -0.0243 | -0.0171 |
| | | (0.0008) | (0.0215)** | (0.0333)** | (0.0393) | (0.0298) |
| 9 | 320 | -0.0008 | 1.0392 | 0.2172 | -0.0178 | 0.0705 |
| | | (0.0011) | (0.0313)** | (0.0613)** | (0.0709) | (0.0499)† |
| 10 | 320 | 0.0007 | 1.0207 | 0.5003 | -0.2430 | 0.0228 |
| | | (0.0013) | (0.0372)** | (0.0795)** | (0.0678)** | (0.0441) |
| Top 10% - | 320 | 0.0038 | -0.0454 | 0.1086 | -0.0592 | 0.0680 |
| Bottom 10% | | (0.0022)* | (0.0714) | (0.0858) | (0.0907) | (0.0470)† |
| Top 20% - | 320 | 0.0008 | -0.0092 | 0.1038 | 0.0332 | 0.1208 |
| Bottom 20% | | (0.0016) | (0.0488) | (0.0656)† | (0.0930) | (0.0440)** |

Table 6: Value Weight Performance Top-Bottom Decile: Small / Medium / Big Stocks

At each quarter-end, we sort stocks into three categories (small, medium and big) according to size and then extract their revealed beliefs. One month into each quarter, for each size category, we sort stocks into deciles based on the difference between the mean of top-30% correlation ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock using the previous four quarters' holding reports. Based on these (BDI) deciles we create value-weight portfolios that we buy at the start of the second month of the quarter. We hold the portfolio for 3 months, repeating the procedure in the subsequent quarter. For each decile, we present the monthly average returns, the excess returns (over the risk free rate), the CAPM, Fama-French, and Carhart alphas, as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). We also provide results for long-short portfolios constructed by buying the top one (two, five) decile(s) and selling the bottom one (two, five) decile(s). Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | Average Return | Excess Return | CAPM Alpha | Fama-French Alpha | Carhart Alpha | DGTW CS |
|---|---|---|---|---|---|---|
| Small | -0.0014 | -0.0014 | -0.0015 | -0.0011 | -0.0033 | -0.0020 |
| | (0.0030) | (0.0030) | (0.0030) | (0.0031) | (0.0034) | (0.0036) |
| Medium | 0.0052 | 0.0052 | 0.0050 | 0.0053 | 0.0037 | 0.0046 |
| | (0.0017)** | (0.0017)** | (0.0017)** | (0.0019)** | (0.0017)* | (0.0019)** |
| Big | 0.0020 | 0.0020 | 0.0019 | 0.0018 | 0.0018 | -0.0004 |
| | (0.0019) | (0.0019) | (0.0020) | (0.0021) | (0.0021) | (0.0015) |

Table 7: Robustness Check using Diagonal Variance-Covariance Matrix: Equal-Weight BDI Decile Stock Portfolios

In this approach, the diagonal terms are estimated using the sample variances of the corresponding stock returns of the last eight quarters. One month into each quarter we sort stocks into deciles based on the difference between the mean of top-30% correlation ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock using the previous four quarters' holding reports. Based on these (BDI) deciles we create equal-weight portfolios that we buy at the start of the second month of the quarter. We hold the portfolio for 3 months, repeating the procedure in the subsequent quarter. For each decile, we present the monthly average returns, the excess returns (over the risk free rate), the CAPM, Fama-French, and Carhart alphas, as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). We also provide results for long-short portfolios constructed by buying the top one (two, five) decile(s) and selling the bottom one (two, five) decile(s). Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | Average Return | Excess Return | CAPM Alpha | Fama-French Alpha | Carhart Alpha | DGTW CS |
|---|---|---|---|---|---|---|
| 1 | 0.0100 | 0.0053 | -0.0045 | -0.0035 | -0.0013 | -0.0014 |
| | (0.0043)$^{**}$ | (0.0043) | (0.0019)$^{*}$ | (0.0011)$^{**}$ | (0.0012) | (0.0011) |
| 2 | 0.0114 | 0.0067 | -0.0014 | -0.0027 | -0.0003 | 0.0000 |
| | (0.0036)$^{**}$ | (0.0036)$^{*}$ | (0.0016) | (0.0010)$^{**}$ | (0.0008) | (0.0008) |
| 3 | 0.0130 | 0.0083 | 0.0011 | -0.0011 | 0.0013 | 0.0008 |
| | (0.0032)$^{**}$ | (0.0032)$^{**}$ | (0.0016) | (0.0010) | (0.0009)$^{\dagger}$ | (0.0008) |
| 4 | 0.0140 | 0.0093 | 0.0024 | 0.0002 | 0.0026 | 0.0015 |
| | (0.0032)$^{**}$ | (0.0032)$^{**}$ | (0.0016)$^{\dagger}$ | (0.0010) | (0.0011)$^{**}$ | (0.0006)$^{**}$ |
| 5 | 0.0145 | 0.0098 | 0.0031 | 0.0009 | 0.0037 | 0.0014 |
| | (0.0030)$^{**}$ | (0.0030)$^{**}$ | (0.0016)$^{*}$ | (0.0011) | (0.0013)$^{**}$ | (0.0007) $^{*}$ |
| 6 | 0.0138 | 0.0091 | 0.0024 | -0.0002 | 0.0021 | 0.0015 |
| | (0.0031)$^{**}$ | (0.0031)$^{**}$ | (0.0017)$^{\dagger}$ | (0.0010) | (0.0010)$^{*}$ | (0.0007) $^{*}$ |
| 7 | 0.0138 | 0.0091 | 0.0025 | -0.0002 | 0.0019 | 0.0012 |
| | (0.0031)$^{**}$ | (0.0031)$^{**}$ | (0.0019)$^{\dagger}$ | (0.0009) | (0.0010)$^{*}$ | (0.0007) $^{*}$ |
| 8 | 0.0134 | 0.0087 | 0.0019 | -0.0007 | 0.0012 | 0.0013 |
| | (0.0032)$^{**}$ | (0.0032)$^{**}$ | (0.0017) | (0.0009) | (0.0009)$^{\dagger}$ | (0.0007) $^{*}$ |
| 9 | 0.0136 | 0.0089 | 0.0016 | -0.0004 | 0.0011 | 0.0009 |
| | (0.0034)$^{**}$ | (0.0034)$^{**}$ | (0.0017) | (0.0008) | (0.0008)$^{\dagger}$ | (0.0006) $^{\dagger}$ |
| 10 | 0.0132 | 0.0084 | -0.0001 | -0.0007 | -0.0002 | 0.0016 |
| | (0.0039)$^{**}$ | (0.0039)$^{*}$ | (0.0019) | (0.0009) | (0.0010) | (0.0008) $^{*}$ |
| Top 10% - | 0.0031 | 0.0031 | 0.0043 | 0.0028 | 0.0011 | 0.0030 |
| Bottom 10% | (0.0011)$^{**}$ | (0.0011)$^{**}$ | (0.0013)$^{**}$ | (0.0011)$^{**}$ | (0.0010) | (0.0009)$^{**}$ |
| Top 20% - | 0.0027 | 0.0027 | 0.0036 | 0.0026 | 0.0013 | 0.0020 |
| Bottom 20% | (0.0009)$^{**}$ | (0.0009)$^{**}$ | (0.0010)$^{**}$ | (0.0009)$^{**}$ | (0.0008)$^{\dagger}$ | (0.0007)$^{**}$ |
| Top 50% - | 0.0010 | 0.0010 | 0.0015 | 0.0008 | 0.0000 | 0.0008 |
| Bottom 50% | (0.0006)$^{\dagger}$ | (0.0006)$^{\dagger}$ | (0.0007)$^{*}$ | (0.0006)$^{\dagger}$ | (0.0007) | (0.0005) $^{*}$ |

Table 8: Robustness Check using Identity Variance-Covariance Matrix: Equal-Weight BDI Decile Stock Portfolios

In this approach, the variance-covariance matrix of the corresponding stock returns is assumed to be an identity matrix. One month into each quarter we sort stocks into deciles based on the difference between the mean of top-30% correlation ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock using the previous four quarters' holding reports. Based on these (BDI) deciles we create equal-weight portfolios that we buy at the start of the second month of the quarter. We hold the portfolio for 3 months, repeating the procedure in the subsequent quarter. For each decile, we present the monthly average returns, the excess returns (over the risk free rate), the CAPM, Fama-French, and Carhart alphas, as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). We also provide results for long-short portfolios constructed by buying the top one (two, five) decile(s) and selling the bottom one (two, five) decile(s). Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | Average Return | Excess Return | CAPM Alpha | Fama-French Alpha | Carhart Alpha | DGTW CS |
|---|---|---|---|---|---|---|
| 1 | 0.0126 | 0.0079 | 0.0004 | -0.0001 | 0.0004 | 0.0001 |
| | (0.0031)** | (0.0031)** | (0.0012) | (0.0007) | (0.0007) | (0.0006) |
| 2 | 0.0132 | 0.0085 | 0.0012 | -0.0001 | 0.0013 | 0.0008 |
| | (0.0030)** | (0.0030)** | (0.0013) | (0.0009) | (0.0008)† | (0.0006) † |
| 3 | 0.0123 | 0.0076 | 0.0002 | -0.0013 | 0.0007 | 0.0002 |
| | (0.0032)** | (0.0032)** | (0.0014) | (0.0009)† | (0.0009) | (0.0006) |
| 4 | 0.0123 | 0.0076 | -0.0001 | -0.0019 | 0.0016 | -0.0000 |
| | (0.0034)** | (0.0034)* | (0.0016) | (0.0013)† | (0.0014) | (0.0008) |
| 5 | 0.0130 | 0.0083 | 0.0003 | -0.0011 | 0.0034 | 0.0003 |
| | (0.0038)** | (0.0038)* | (0.0021) | (0.0017) | (0.0022)† | (0.0011) |
| 6 | 0.0127 | 0.0080 | -0.0002 | -0.0017 | 0.0030 | 0.0015 |
| | (0.0040)** | (0.0040)* | (0.0023) | (0.0016) | (0.0019)† | (0.0013) |
| 7 | 0.0124 | 0.0077 | 0.0000 | -0.0020 | 0.0013 | 0.0012 |
| | (0.0036)** | (0.0036)* | (0.0019) | (0.0011)* | (0.0012) | (0.0008) † |
| 8 | 0.0130 | 0.0083 | 0.0011 | -0.0007 | 0.0007 | 0.0012 |
| | (0.0032)** | (0.0032)** | (0.0017) | (0.0009) | (0.0008) | (0.0006) * |
| 9 | 0.0136 | 0.0089 | 0.0018 | 0.0003 | 0.0007 | 0.0013 |
| | (0.0030)** | (0.0030)** | (0.0016) | (0.0007) | (0.0007) | (0.0005)** |
| 10 | 0.0148 | 0.0101 | 0.0030 | 0.0020 | 0.0009 | 0.0022 |
| | (0.0031)** | (0.0031)** | (0.0015)* | (0.0007)** | (0.0007)† | (0.0004)** |
| Top 10% - | 0.0022 | 0.0022 | 0.0025 | 0.0021 | 0.0004 | 0.0020 |
| Bottom 10% | (0.0009)** | (0.0009)** | (0.0011)** | (0.0010)* | (0.0008) | (0.0007)** |
| Top 20% - | 0.0013 | 0.0013 | 0.0016 | 0.0013 | -0.0001 | 0.0012 |
| Bottom 20% | (0.0008)† | (0.0008)† | (0.0009)* | (0.0009)† | (0.0007) | (0.0006) * |

Table 9: Robustness Check using Return Gap: Equal-Weight BDI-Decile Stock Portfolios

We restrict our sample by excluding funds which are ranked top 30% in terms of the return gap (Kacperczyk, Sialm, and Zheng (2008)) in absolute terms. One month into each quarter we sort stocks into deciles based on the difference between the mean of top-20% correlation ranked managers' normalized beliefs and the mean of the remaining managers' normalized beliefs for the stock using the previous quarter's holding reports. Based on these (BDI) deciles we create equal-weight portfolios that we buy at the start of the second month of the quarter. We hold the portfolio for 3 months, repeating the procedure in the subsequent quarter. For each decile portfolio, we present the monthly average returns, the excess returns (over the risk free rate), the CAPM, Fama-French, and Carhart alphas, as well as the characteristic selectivity (CS) measure of Daniel, Grinblatt, Titman, and Wermers (1997). We also provide results for long-short portfolios constructed by buying the top one (two, five) decile(s) and selling the bottom one (two, five) decile(s). Decile 1 (Decile 10) contains the lowest (highest) BDI-ranked stocks.

| Decile | Average Return | Excess Return | CAPM Alpha | Fama-French Alpha | Carhart Alpha | DGTW CS |
|---|---|---|---|---|---|---|
| 1 | 0.0103 | 0.0056 | -0.0032 | -0.0042 | 0.0001 | -0.0010 |
| | (0.0039)$^{**}$ | (0.0039)$^{\dagger}$ | (0.0020)$^{*}$ | (0.0016)$^{**}$ | (0.0016) | (0.0013) |
| 2 | 0.0121 | 0.0074 | -0.0002 | -0.0019 | 0.0006 | 0.0005 |
| | (0.0032)$^{**}$ | (0.0032)$^{*}$ | (0.0016) | (0.0012)$^{\dagger}$ | (0.0010) | (0.0008) |
| 3 | 0.0131 | 0.0084 | 0.0012 | -0.0008 | 0.0011 | 0.0007 |
| | (0.0030)$^{**}$ | (0.0030)$^{**}$ | (0.0015) | (0.0011) | (0.0009) | (0.0007) |
| 4 | 0.0142 | 0.0095 | 0.0026 | 0.0006 | 0.0020 | 0.0019 |
| | (0.0029)$^{**}$ | (0.0029)$^{**}$ | (0.0014)$^{*}$ | (0.0009) | (0.0009)$^{**}$ | (0.0006)$^{**}$ |
| 5 | 0.0137 | 0.0090 | 0.0022 | 0.0000 | 0.0010 | 0.0011 |
| | (0.0028)$^{**}$ | (0.0028)$^{**}$ | (0.0014)$^{\dagger}$ | (0.0009) | (0.0008) | (0.0006) $^{*}$ |
| 6 | 0.0136 | 0.0089 | 0.0019 | -0.0002 | 0.0010 | 0.0007 |
| | (0.0029)$^{**}$ | (0.0029)$^{**}$ | (0.0014)$^{\dagger}$ | (0.0008) | (0.0008) | (0.0006) |
| 7 | 0.0130 | 0.0083 | 0.0013 | -0.0008 | 0.0005 | 0.0002 |
| | (0.0029)$^{**}$ | (0.0029)$^{**}$ | (0.0014) | (0.0009) | (0.0008) | (0.0006) |
| 8 | 0.0144 | 0.0096 | 0.0025 | 0.0005 | 0.0021 | 0.0012 |
| | (0.0029)$^{**}$ | (0.0029)$^{**}$ | (0.0014)$^{*}$ | (0.0008) | (0.0009)$^{**}$ | (0.0006) $^{*}$ |
| 9 | 0.0126 | 0.0079 | 0.0004 | -0.0016 | 0.0006 | 0.0007 |
| | (0.0031)$^{**}$ | (0.0031)$^{**}$ | (0.0015) | (0.0010)$^{*}$ | (0.0010) | (0.0007) |
| 10 | 0.0129 | 0.0081 | -0.0004 | -0.0007 | 0.0028 | 0.0008 |
| | (0.0038)$^{**}$ | (0.0038)$^{*}$ | (0.0018) | (0.0012) | (0.0014)$^{*}$ | (0.0011) |
| Top 10% - | 0.0025 | 0.0025 | 0.0028 | 0.0035 | 0.0027 | 0.0019 |
| Bottom 10% | (0.0011)$^{*}$ | (0.0011)$^{*}$ | (0.0012)$^{**}$ | (0.0012)$^{**}$ | (0.0012)$^{*}$ | (0.0011) $^{*}$ |
| Top 20% - | 0.0015 | 0.0015 | 0.0017 | 0.0019 | 0.0014 | 0.0010 |
| Bottom 20% | (0.0009)$^{*}$ | (0.0009)$^{*}$ | (0.0010)$^{*}$ | (0.0010)$^{*}$ | (0.0010)$^{\dagger}$ | (0.0008) $^{\dagger}$ |