

Aggregate Stock Market Risk Premia and Real Economic Activity

Antonio Mele
London School of Economics

January 20, 2008

Very preliminary draft. This is still work in progress

Abstract

This paper provides novel empirical evidence about the cyclical behavior of the equity premium in the US. I find that the equity premium exhibits a quite pronounced asymmetric pattern, in that it increases more during recessions than it decreases during expansions. I calculate that on average, the equity premium is 80% larger during recessions than during expansions. I also uncover new facts about the predictability of real economic activity. I find that the equity premium is able to explain up to 20% of the average drop in growth of industrial production occurring during recessions.

1. Introduction

This paper uncovers new empirical evidence about the pattern of the equity premium over the business cycle. I find that this pattern is asymmetric, in that upward movements occurring during recessions are far more severe than downward movements occurring during expansions. Moreover, I find evidence that the equity premium explains a significant portion of the business cycle development. More generally, I estimate a dynamic model in which the equity premium and the business cycle affect each other, and find that the feedbacks are both statistically and economically important.

Our interest in the asymmetric behavior of the equity premium stems from recent theoretical research about stock market *volatility*. Financial economists have known for decades that stock volatility is countercyclical, being larger in bad times than in good times (see, e.g., Mele (2008) for a survey). However, it is only recently that theoretical models have been produced, which might potentially be consistent with this empirical regularity (e.g., Basak and Cuoco (1998), Campbell and Cochrane (1999), Menzly, Santos and Veronesi (2004), Mele (2007)). What is, then, the connection between stock market volatility and the equity premium? In a companion paper (see Mele (2007)), I show a quite general theoretical result that in frictionless markets, countercyclical stock volatility occurs if the equity premium (i.e. the compensation required by investors in the aggregate equity market) increases more in bad times than it decreases in good times, thereby inducing stock prices to be more volatile in bad times than in good. In other words, an asymmetric behavior of the equity premium over the business cycle can be responsible of countercyclical stock market volatility.¹ The present paper thus aims to investigate whether the empirical evidence is supportive of the previous theoretical prediction about the equity premium. Figure 1 summarizes informal pieces of evidence in support of this prediction. The figure reveals that the equity premium is largely countercyclical, which confirms a finding well-known in the literature (see Fama and French (1989) and Ferson and Harvey (1991)). At the same time, this figure reveals that large swings in the equity premium occur during recessions, a fact which I aim to investigate in detail in this paper.

The basic empirical evidence in Figure 1 needs to be further qualified. It might be possible that the equity premium behaves asymmetrically over the business cycle, as a result of complex short-run adjustments. More specifically, the equity premium might increase in response to shocks that worsen the health of the economy. But an increased equity premium can worsen the firms financial constraints, due for example to capital markets imperfections. Therefore, the equity premium might actually work as an amplification mechanism. To model these dynamic effects, I consider a model

¹More formally, this companion paper shows that if the risk premium is sufficiently asymmetric over the business cycle, the price-dividend ratio is then an increasing and *concave* function of variables tracking the business cycle conditions. It is this concavity feature of the price-dividend ratio to make return volatility increase on the downside.

in which the equity premium is explicitly driven by past shocks in some state variable tracking the business cycle conditions. To keep the model as parsimonious as possible, I use the real industrial production index growth (hereafter, “real growth”) as a proxy for such a state variable. In this model, the equity premium is allowed to respond asymmetrically to past shock in real growth. More in detail, I consider a model in which the equity premium response function to past shocks in the real growth is piecewise linear. I find that on average, and in the short-run, a negative shock in the real growth is followed by an increase in the equity premium which is almost *twice* as its decline occurring after a positive shock of the same size. These results remain robust after controlling for the traditional variables known to affect the cyclical properties of the equity premium.

The second distinctive feature of the models in this article is an explicit consideration of feedback effects, or how risk-premia are related to the development of the business cycle. It is well-known that (i) a large fraction of stock market returns can be explained by future movements of real activity; and (ii) the real growth is indeed correlated with past stock market returns (see Fama (1990) and Schwert (1990)). In this paper, instead, I do not consider the issue of predictability of economic activity through past realized returns. Rather, I consider how economic activity is related to the equity premium. In other words, this paper does not investigate the relation between economic activity and past *realized returns*, but the relation between economic activity and *expected returns*. This issue is economically very important, as risk-premia obviously encode information about the market expectation and the necessary risk-corrections related to the future prospects of the economy. In a nutshell, I find that the equity premium is negatively and related to future real growth, one year. At the same time, I find that the real growth affects the future equity premium more than the equity premium affects future real growth.

The paper is organized in the following manner. In the next section, I provide a literature review. In Section 3, I discuss the modeling approach. In Section 4, I provide the results. Section 5 concludes.

2. Literature review

2.1. Expected returns

2.2. Volatility

2.3. The stock market and real economic activity

3. Modeling the cyclical behavior of the equity premium

3.1. Expected returns

I consider a model in which the expected excess returns at time t , denoted as \mathcal{E}_{t+1} , depend on their own past and a number of additional variables z_t , viz

$$\mathcal{E}_t \equiv E(\text{Exc}_t | F_{t-1}) = f(\mathcal{E}_{t-1}, \dots, \mathcal{E}_{t-p}, z_{t-1}; \phi),$$

where f is some function, F_t is the information set as of time t , Exc_t are the realized returns at time t , and ϕ is a parameter vector. Thus, \mathcal{E}_{t+1} is the equity premium as of time t . This formulation simply reduces to updating a conditional expectation with new information. The first basic model will elaborate on a formulation in which $f(\cdot)$ is linear,

$$\Phi(L)\mathcal{E}_t = a_1 \text{Ind}_{t-1} + a_2 \sigma_{t-1}, \tag{1}$$

where $\Phi(L)$ is some polynomial lag operator, Ind_t is the seasonally adjusted industrial production growth as of month t , σ_t is the excess return volatility as of month t , and a_1 and a_2 are some constants. This formulation differs from those proposed in the empirical literature, as I am explicitly using macroeconomic variables to model the equity premium dynamics. However, in the empirical section, I shall control for the significance of the estimates, using traditional predictors of the equity premium.

3.2. Volatility

I aim to simultaneously modelling expected returns and volatility. For this reason, I use ARCH models, as they are relatively simple to implement when one wishes to model complex feedbacks and interactions, as I do here. (See Engle (2000) for the classical papers on ARCH modeling.)

3.3. Industrial production

I assume that the real production growth rate is autoregressive, and heteroskedastic. Following Fama (1990), I also assume that past realized excess returns affect the real growth rate. Moreover, the model I consider is one in which the real growth rate is affected by both past returns and past *expected* returns.

4. Results

4.1. Preliminary results

In Table 3, I report preliminary results, related to the estimation of the following model:

$$\text{Exc}_t = \phi + \epsilon_{1t} \quad ; \quad \text{Ind}_t = a_0 + a_1 \text{Ind}_{t-1} + \sum_{i=1}^4 b_i \text{Exc}_{t-i} + \epsilon_{2t}, \quad (2)$$

where $(a_i)_{i=0,1}$ and $(b_i)_{i=1}^4$ are the parameters to be estimated. These parameters are estimated under two assumptions related to heteroskedasticity.

Under the first assumption, ϵ_{1t} and ϵ_{2t} are independent and normally distributed with variance σ_z^2 and σ_y^2 , respectively, where σ_z^2 and σ_y^2 are to be estimated.

Under the second assumption,

$$\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \mid F_{t-1} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{1t}^2 & \rho \sigma_{1t} \sigma_{2t} \\ \rho \sigma_{1t} \sigma_{2t} & \sigma_{2t}^2 \end{bmatrix} \right) \quad (3)$$

where,

$$\begin{aligned} \sigma_{1t} &= \omega_1 + \alpha_1 |\epsilon_{1t-1}| + \beta_1 \sigma_{1t-1} + \theta_1 \epsilon_{1t-1} \\ \sigma_{2t} &= \omega_2 + \alpha_2 |\epsilon_{2t-1}| + \beta_2 \sigma_{2t-1} + \theta_2 \epsilon_{2t-1} \end{aligned}$$

and $(\omega_i, \alpha_i, \beta_i, \theta_i)_{i=1,2}$ are parameters to be estimated.

4.2. The equity premium and the business cycle

Table 4 presents maximum likelihood estimates of the model,

$$\begin{bmatrix} \text{Exc}_t \\ \text{Ind}_t \end{bmatrix} = \begin{bmatrix} \mathcal{E}_t \\ a_0 + a_1 \text{Ind}_{t-1} + a_2 \mathcal{E}_{t-1} + \sum_{i=1}^4 b_i \cdot \text{Exc}_{t-i} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \quad (4)$$

where

$$\begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix} \mid F_{t-1} \sim N \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{1t}^2 & \rho\sigma_{1t}\sigma_{2t} \\ \rho\sigma_{1t}\sigma_{2t} & \sigma_{2t}^2 \end{bmatrix} \right), \quad (5)$$

$$\begin{cases} \sigma_{1t} = \omega_1 + \alpha_1 |\epsilon_{1t-1}| + \beta_1 \sigma_{1t-1} + \theta_1 \epsilon_{1t-1} \\ \sigma_{2t} = \omega_2 + \alpha_2 |\epsilon_{2t-1}| + \beta_2 \sigma_{2t-1} + \theta_2 \epsilon_{2t-1} + \theta_3 \mathcal{E}_{t-1} \\ \mathcal{E}_t - \mathcal{E}_{t-1} = \phi_0 + \phi_{1a} \mathcal{E}_{t-1} + \phi_{1c} \sigma_{1t-1} + g(\epsilon_{2t-1}) \end{cases} \quad (6)$$

and

$$g(\epsilon) \equiv -\frac{1}{2}\phi_{2a} (|\epsilon| + \epsilon) + \frac{1}{2}\phi_{2b} (|\epsilon| - \epsilon). \quad (7)$$

One important feature of this model is that it allows expected returns \mathcal{E} to react asymmetrically to past shocks. For example, suppose that $\phi_{2a} < \phi_{2b}$. Then, positive variations in expected returns following a negative shock ϵ are higher in absolute value than negative variations in expected returns following a positive shock ϵ of the same size. The reaction function (7) is the key specification that allows us to investigate more thoroughly the asymmetric behavior of the equity premium over the business cycle.

The second key specification of this model is that it allows expected returns \mathcal{E} to feed back both the first and the second moment of the industrial production growth.

Also reported is an estimate of $\rho_{\mathcal{E},\sigma_1}$, the conditional correlation between the equity premium \mathcal{E}_t and returns volatility σ_{1t} .

Figure 2 depicts the estimated risk-premium. I note the following facts. The equity premium is always positive. Its range of variation is higher during the 70s. The estimated expected returns start to rise well before a NBER-dated recession. They decrease after a NBER-dated recession. Naturally, this is partly due to the very specification of the model, by which expected returns are essentially explained through the same very variables which are used to dating a recession (the industrial production index). However, the model also predicts episodes of high expected returns occurring when the global economic outlook and, hence, the industrial production growth rate, was quite good. For example, the model predicts a rise in the expected returns occurring during 1987 crash. Another interesting property is that the equity premium seems to achieve approximately the same value at each NBER-dated peak (12%, on average). The only exception is the period related to the 70s, where this level was almost twice the average.

Evidence of asymmetry. The Wald test of $H_o : \phi_{2a} - \phi_{2b} = 0$ rejects the null at the conventional 95% level, and the results are economically significant, as shown in Figure 3, which depicts the model-based counterpart of Figure 1.

The model makes also predictions about the shape of the conditional correlation of volatility and the equity premium. This shape is hump shape. The *conditional* correlation between the market risk-premium and return volatility, $\rho_{\sigma_y, \mathcal{E}}$, is simply the *unconditional* correlation between the error terms, which I can estimate as the coefficient of correlation between $\mathcal{E}_t - \mathcal{E}_{t-1}$ and $\sigma_{y,t} - \sigma_{y,t-1}$. I estimate $\rho_{\sigma_y, \mathcal{E}}$ to be -0.166 , and find that this estimate is significant. These results are consistent with previous results by Brandt and Kang (2002, J. Mon. Econ.), but they are obtained with the help of macroeconomic data, which suggests that the empirical evidence in Brandt and Kang might interpreted as one in which their latent factors were a proxy for the business cycle.

Table 5 presents basic statistics for one year equity premium, stock returns volatility as predicted by model (4)-(6).

According to this model, a typical increase in the risk-premium during a NBER-dated recession (i.e. 2.5%) can account for an average of approximately 20% drop in the industrial production rate during a typical recession, and for an average of approximately 6% increase in the industrial production rate volatility during a typical recession.²

Finally, the model generates quite high conditional correlations between risk-premia and industrial production, of the order of -75% . See Figure 4. The shape is interestingly asymmetric. Of course, stock returns also contribute to the story, but on a quantitative standpoint, their contribution is half the contribution of risk-premia.

4.3. The traditional, financial view of stock market risk-premia

Next, I develop a measure of the equity premium related to corporate default and the slope of the term structure. As originally demonstrated by Fama and French (1989), these variables display a clear business cycle pattern and therefore represent an ideal competitor to the model in the previous section. However, Fama (1990 p. 1101) also observed that when industrial production growth rates are used to explain *realized* returns, term-spread (TERM) effects evaporate. Fama attributed this fact to TERM to be *positively* related to industrial production growth rates. Further details on TERM

²These figures are computed as follows: During recessions, \mathcal{P} increases by approximately 2.5%. And a 2.5% increase in \mathcal{P} corresponds roughly to $\frac{2.5}{12}\%$ in \mathcal{E} . According to the multipliers of model ?, a $\frac{2.5}{12}\%$ increase in \mathcal{E} translates to a $\frac{2.5}{12}\theta_3\%$ increase in σ_2 which annualized yields a percentage increase of $\sqrt{12}\frac{2.5}{12}\theta_3$. During recessions, variations in the industrial production growth volatility take on an average of 0.093 (see Table 5). Therefore, this model predicts that the percentage of positive variations in the industrial production growth volatility explained by changes in the risk-premium is equal to $\frac{\sqrt{12}\frac{2.5}{12}\theta_3}{0.093} = 6.208\%$. Similarly, the model predicts that the percentage of negative variations in industrial production growth explained by changes in the risk-premium is equal to $\frac{\frac{2.5}{12}a_2}{0.748} = 19.079\%$, where -0.748 is the average of negative variations in the industrial production growth during recessions (see Table 2, Panel A).

are in Fama and French (1989). The perspective, here, is different, as I am investigating how these macroeconomic variables affect the *expected* returns. In the context of this paper, it is natural to compare the two equity premia that emerge when I take into account traditional exogeneous predictors and macroeconomic predictors.

I define the term premium as $\text{term}_t \equiv (Aaa/12 - r_f)$ and the default premium as $\text{def}_t \equiv (Baa - Aaa)/12$, where *Aaa* and *Baa* are the (percentage) Moody's Baa and Moody's Aaa annual corporate bond yields. These are the exogeneous predictors that are typically used (see, e.g., the above authors).

I regress y_t on to lagged values of default premium and term premium,

$$\text{Exc}_t = \text{const.} + \gamma_1 \cdot \text{def}_{t-1} + \gamma_2 \cdot \text{term}_{t-1} + u_t, \quad (8)$$

where u_t is a residual term.

In Figure 4, I compare the two $E_t(\mathcal{E}_{t+12})$ that are predicted by model (5) and the simple model (8). The only pieces of information that I used to compute macroeconomic risk-premia were stock returns and production growth rates. Yet the two risk-premia display a similar pattern of variation. The large swings during the 1970s and the beginning of the 1980s are episodes that both models capture. The purely financial model seems to fail to capture other episodes captured by the macroeconomic model (e.g., the quite modest rise in the risk-premia at the beginning of the 1970s). More generally, risk-premia generated this way do not keep track *directly* of the general business conditions. On the other hand, the model in the previous section could be restrictive as well, since it simply ignores some term-structure information. Moreover, it is important to see whether some of the results in the previous section are robust, once we control for the traditional financial indicators.

4.4. Unified approach

I just replace the risk-premium equation in (6) with

$$\mathcal{E}_t - \mathcal{E}_{t-1} = \phi_0 + \phi_{1a}\mathcal{E}_{t-1} + \phi_{1c}\sigma_{1t-1} + g(\epsilon_{t-1}^z) + \gamma_1 \cdot \text{def}_{t-1} + \gamma_2 \cdot \text{term}_{t-1}. \quad (9)$$

Table 6 reports maximum likelihood estimates for this augmented model. The results in the previous section seem to be quite robust to this more general specification. The relation between risk-premia and the default variable is still positive, but contrary to Fama (1990), I find that the risk-premium is negatively related to the term variable. While the coefficients are significant, a likelihood ratio tests fails to reject the macroeconomic model with a p -value of 0.781. The only two things that seem to change significantly are 1) the gap $\phi_{2a} - \phi_{2b}$ (test =), and that θ_3 increases a lot. Otherwise,

the evidence is that overall, exogeneous predictors do not seem to alter the results discussed in the previous section. Indeed, the cyclical properties of the equity premium are the same. The time series of the estimated equity premium is very similar to that depicted in Figure 2. The gap $\phi_{2a} - \phi_{2b}$ (test =) shrinks, but this is because I am also using as a proxy something (def) which behaves asymmetrically in the first place (see Figure 1 + produce some formal tests to support this claim.). But it is still interesting that the gap $\phi_{2a} - \phi_{2b}$ persists, and that it is economically important.

5. Conclusions

- (i) The equity premium reacts asymmetrically to shocks in the real industrial production growth. It increases more after a negative shock than it decreases after a positive shock.
- (ii) The volatility of the industrial production growth is negatively affected by the equity premium required in the past.
- (iii) There is no evidence that the equity premium affects stock market volatility.
- (iv) The equity premium is positively affected by past stock market volatility.

References

- Basak, S., Cuoco, D., 1998. "An equilibrium model with restricted stock market participation." *Review of Financial Studies* 11, 309-341.
- Campbell, J.Y., and J.H. Cochrane, 1999. "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy* 107, 205-251.
- Engle, R.F., 2000. *ARCH: Selected Readings*. Oxford: Oxford University Press.
- Fama, E.F., 1990. "Stock Returns, Expected Returns, and Real Activity." *Journal of Finance* 45 1089-1108.
- Fama, E. F., K.R. French, 1989. "Business Conditions and Expected Returns on Stock and Bonds." *Journal of Financial Economics* 25, 23-49.
- Ferson, W.E. and C.R. Harvey, 1991. "The Variation of Economic Risk Premiums." *Journal of Political Economy* 99, 385-415.
- Mele, A., 2007. "Asymmetric Stock Market Volatility and the Cyclical Behavior of Expected Returns." *Journal of Financial Economics* 86, 446-478.
- Mele, A., 2008. "Understanding Stock Market Volatility: A Business Cycle Perspective." Working paper, London School of Economics.
- Menzly, L., T. Santos and P. Veronesi, 2004. "Understanding Predictability." *Journal of Political Economy* 111, 1-47.
- Schwert, G.W., 1989. "Why Does Stock Market Volatility Change Over Time?" *Journal of Finance* 44, 1115-1153.
- Schwert, G.W., 1990. "Stock Returns and Real Activity: A Century of Experience." *Journal of Finance* 45, 1237-1257.

Tables

Table 1 – Business cycle properties of P/D ratios and returns*

	total		NBER expansions		NBER recessions	
	average	std dev	average	std dev	average	std dev
P/D	31.999	15.878	33.213	15.796	26.204	14.888
$P/D_{t+1} - P/D_t$	0.054	1.293	0.110	1.202	-0.211	1.627
$\ln \frac{P/D_{t+1}}{P/D_t} \times 12 \times 100$	2.008	42.023	3.954	37.442	-7.279	58.165
returns (real)	8.224	51.778	9.702	47.859	1.175	66.789
smooth returns (real)	8.456	15.787	12.192	13.056	-9.372	15.445
risk-free rate (nominal)	4.815	2.771	4.534	2.373	6.158	3.901
risk-free rate (real)	1.156	2.247	1.185	2.146	1.016	2.662
excess returns	7.133	51.408	8.643	47.567	-0.075	66.112
smooth excess returns	7.438	15.448	11.165	12.652	-10.345	15.130

*P/D is the S&P Comp. price-dividend ratio. Smooth returns as of time t are defined as $\sum_{i=1}^{12} (\text{exc}_{t-i} - R_{t-i})$, where exc_t is the S&P Comp. return, and R is the risk-free rate. Volatility is the excess returns volatility. With the exception of the P/D ratio levels and variations, all figures are annualized percent. Data are sampled monthly and cover the period from January 1948 through December 2002.

Table 2 – Seasonally adjusted industrial production growth*

Panel A: Basic statistics

	total		NBER expansions		NBER recessions	
	average	std dev	average	std dev	average	std dev
Ind	0.278	1.028	0.493	0.898	-0.748	0.982
IP	0.282	0.486	0.391	0.428	-0.238	0.398

Panel B: Estimate and standard deviation of the autocorrelation function

lag	1	2	3	4	5	6	7	8	9	10
estimate	0.399	0.250	0.182	0.113	0.022	0.006	0.0453	0.0389	0.003	-0.011
std dev	0.038	0.044	0.046	0.047	0.048	0.048	0.048	0.048	0.048	0.048

*This table presents basic statistics for monthly industrial production rates. Ind_t is the real, seasonally adjusted US industrial production growth rate; and IP is the moving average real industrial production growth rate defined as $IP_t = (Ind_t + \dots + Ind_{t-11})/12$. Panel A contains basic statistics and Panel B contains the estimated autocorrelation function of industrial production growth Ind.

Table 3 – Maximum Likelihood estimates of model (2)*

Parameter	Homoskedastic		Heteroskedastic	
	estimate	std dev	estimate	std dev
a_0	0.125	0.038	0.157	0.021
a_1	0.371	0.048	0.303	0.027
b_1	0.014*	0.008	0.006*	0.004
b_2	0.024	0.007	0.018	0.006
b_3	0.020	0.007	0.007*	0.005
b_4	0.021	0.007	0.017	0.006
ϕ	0.594	0.166	0.621	0.152
σ_1	4.281	0.085	—	—
σ_2	0.924	0.051	—	—
ω_1	—	—	0.641	0.016
α_1	—	—	0.097	0.005
β_1	—	—	0.771	0.004
θ_1	—	—	-0.089	0.017
ω_2	—	—	0.064	0.003
α_2	—	—	0.185	0.005
β_2	—	—	0.791	0.004
θ_2	—	—	-0.111	0.012
ρ	—	—	0.011*	0.035

*The column labeled “Homoskedastic” reports results of the model obtained under the assumption that ε_{1t} and ε_{2t} are independent, and normal with variance σ_1^2 and σ_2^2 , respectively. The column labeled “Heteroskedastic” reports results of the model obtained under the ARCH assumption (3). For each model, the first column reports coefficient estimates and the second column reports robust standard errors of the estimates. Starred figures refer to estimates that are not statistically different from zero at the 5% level. The log-likelihood function evaluated at the estimated parameter is -1563.304 (“Homoskedastic”) and -1450.407 (“Heteroskedastic”).

Table 4 – Maximum likelihood estimates of model (4)-(6)*

	estimate	std dev		estimate	std dev
a_0	0.627	0.024	ϕ_0	-0.183	0.007
a_1	0.201	0.029	ϕ_{1a}	-0.203	0.011
a_2	-0.685	0.035	ϕ_{1c}	0.074	0.002
ω_1	1.586	0.028	ϕ_{2a}	0.108	0.022
α_1	0.099	0.009	ϕ_{2b}	0.186	0.027
β_1	0.539	0.007	b_1	0.007*	0.003
θ_1	-0.158	0.022	b_2	0.018	0.006
ω_2	0.030	0.002	b_3	0.005*	0.005
α_2	0.163	0.004	b_4	0.008*	0.006
β_2	0.844	0.003	ρ	0.025*	0.037
θ_2	-0.107	0.011	$\rho_{\mathcal{E},\sigma_1}$	-0.166	0.069
θ_3	0.008	0.002			

*Starred figures refer to estimates that are not statistically different from zero at the 5% level. $\rho_{\mathcal{E},\sigma_1}$ is an estimate of the conditional correlation between equity premium \mathcal{E}_t and returns volatility σ_{1t} . The log-likelihood function evaluated at the estimated parameter is -1427.305.

Table 5 – Business cycle properties of equity premium and macroeconomic volatility: Model’s prediction*

	total		NBER expansions		NBER recessions	
	average	std dev	average	std dev	average	std dev
equity premium (\mathcal{P})	8.407	2.260	7.749	1.646	11.545	2.142
$\mathcal{P}_t - \mathcal{P}_{t-1}$	0.003	1.023	-0.053	0.989	0.277	1.125
$\ln \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} \times 100$	0.037	12.452	-0.482	12.840	2.516	9.970
returns volatility (σ_1)	14.512	3.537	13.901	2.987	17.432	4.384
$\sigma_{1t} - \sigma_{1t-1}$	0.019	2.961	0.052	2.726	-0.137	3.876
$\ln \frac{\sigma_{1t}}{\sigma_{1t-1}} \times 100$	0.102	17.568	0.359	16.893	-1.121	20.373
IP volatility (σ_2)	2.943	1.245	2.752	1.113	3.857	1.418
$\sigma_{2t} - \sigma_{2t-1}$	-0.002	0.454	-0.022	0.410	0.093	0.613
$\ln \frac{\sigma_{2t}}{\sigma_{2t-1}} \times 100$	-0.119	13.391	-0.712	12.609	2.708	16.277

*This table presents basic statistics for one year equity premium, stock returns volatility, and industrial production volatility (Panel C), as predicted by model (1). The one year equity premium as of time t is computed as $\mathcal{P}_t \equiv \sum_{i=1}^{12} E(\mathcal{E}_{t+i} | F_t)$, where \mathcal{E}_{t+i} is generated by model (4)-(6), and $E(\cdot | F_t)$ denotes the conditional expectation given F_t , the information set as of time t . The two volatilities σ_{1t} and σ_{2t} are also obtained from the estimated model (4)-(6), and have both been rescaled by a factor of $\sqrt{12}$. Conditional expectations are computed through Monte Carlo integration (100 repetitions for every single point).

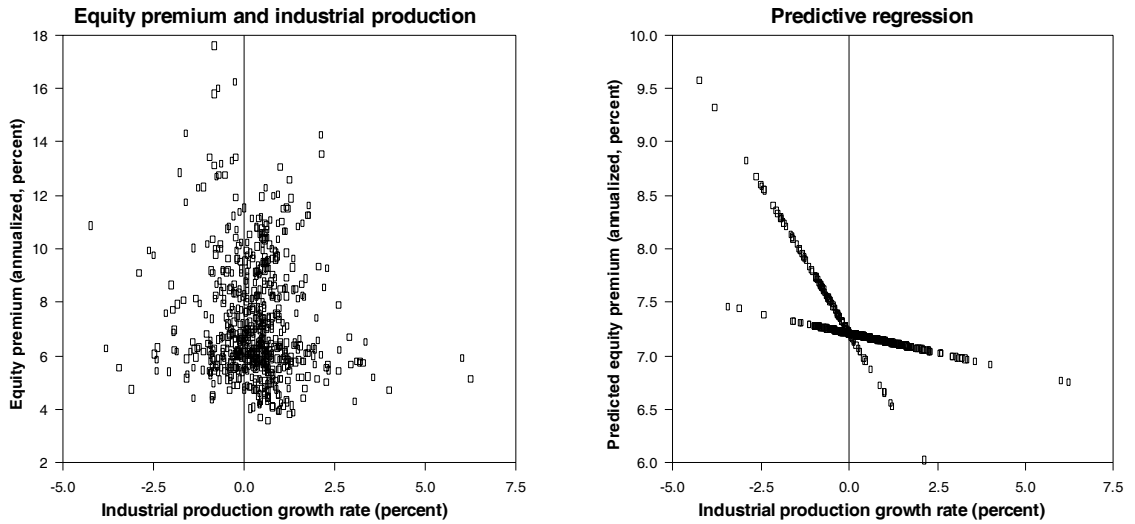
Table 6 – Augmented model*

	<u>estimate</u>	<u>std dev</u>		<u>estimate</u>	<u>std dev</u>
a_0	0.671	0.025	ϕ_{1a}	-0.214	0.012
a_1	0.181	0.027	ϕ_{1c}	0.080	0.002
a_2	-0.739	0.035	ϕ_{2a}	0.107	0.022
ω_1	1.554	0.027	ϕ_{2b}	0.141	0.031
α_1	0.093	0.008	b_1	0.007*	0.004
β_1	0.552	0.007	b_2	0.019	0.006
θ_1	-0.158	0.020	b_3	0.004*	0.005
ω_2	0.025	0.002	b_4	0.008*	0.006
α_2	0.179	0.005	ρ	0.019*	0.037
β_2	0.822	0.003	$\rho_{\mathcal{E},\sigma_1}$	-0.189	0.069
θ_2	-0.101	0.011	γ_1	0.373	0.098
θ_3	0.027	0.003	γ_2	-0.199	0.038
ϕ_0	-0.179	0.007			

*This table presents maximum likelihood estimates of model (4)-(6), with the risk-premium specification augmented to control for default premium and term-premium variables. Starred figures refer to estimates that are not statistically different from zero at the 5% level. The log-likelihood function evaluated at the estimated parameter is -1425.785 .

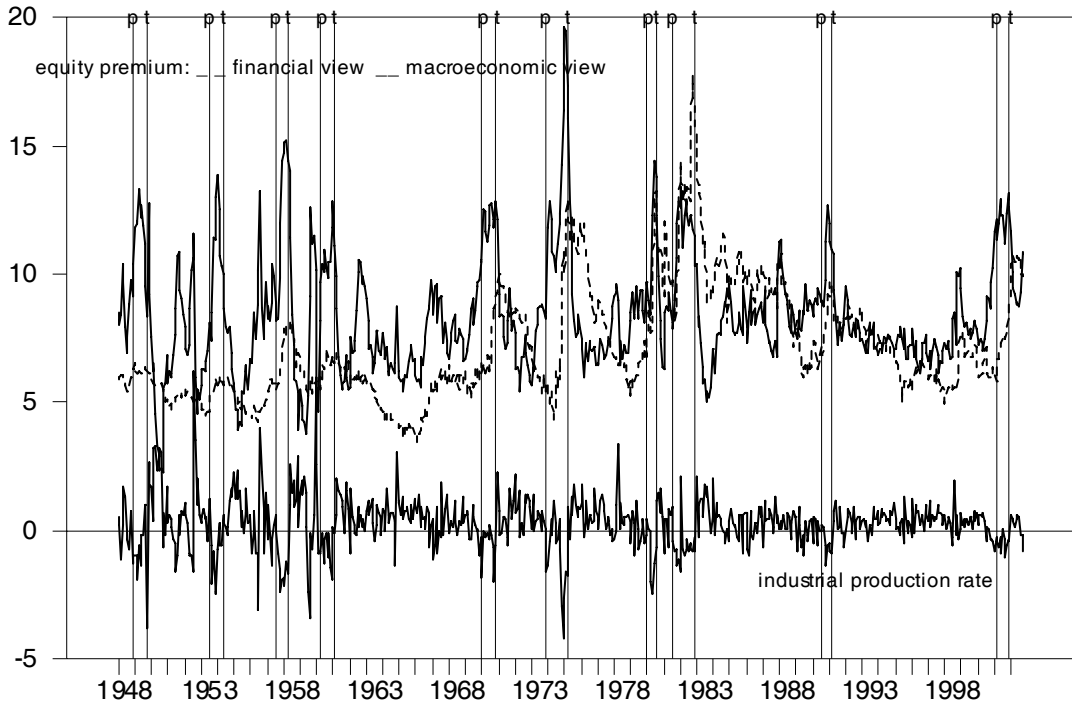
Figures

Figure 1 – Equity premium and economic conditions



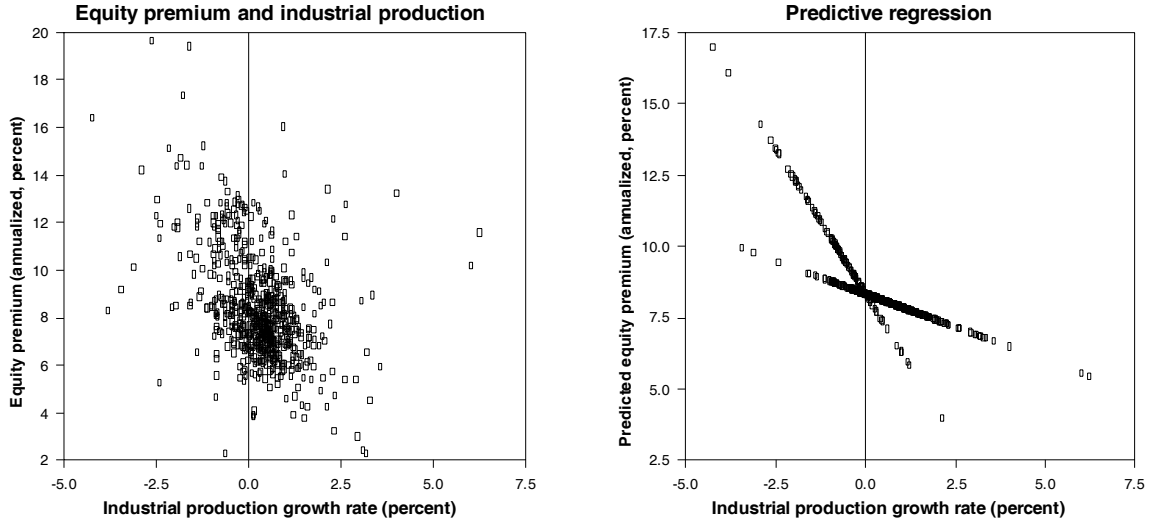
*The left hand side panel of this picture plots expected excess returns $\hat{\mathcal{E}}_t$ (say) against the real, monthly, seasonally adjusted US industrial production growth rate. (Data span and frequency are as in Tables 1 and 2.) The series on the expected excess returns, $\hat{\mathcal{E}}_t$, is obtained through Fama and French (1989) predictive regression of S&P returns on to the default-premium and the term premium. The predictive regression depicts the prediction of the Ordinary Least Squares regression: $\hat{\mathcal{E}} = \underset{(0.098)}{7.212} - \mathbb{I}_{\text{recession}} \cdot \underset{(0.209)}{0.556} \cdot \text{Ind} - \mathbb{I}_{\text{expansion}} \cdot \underset{(0.094)}{0.073} \cdot \text{Ind} + w$, where $\mathbb{I}_{\text{recession}}$ (resp. $\mathbb{I}_{\text{expansion}}$) is the indicator function taking the value one if the economy is in a NBER-recession (resp. expansion) episode and zero otherwise, w is a residual term, and standard errors are in parenthesis.

Figure 2 – One year ahead equity premium estimated through model (4)-(6)*



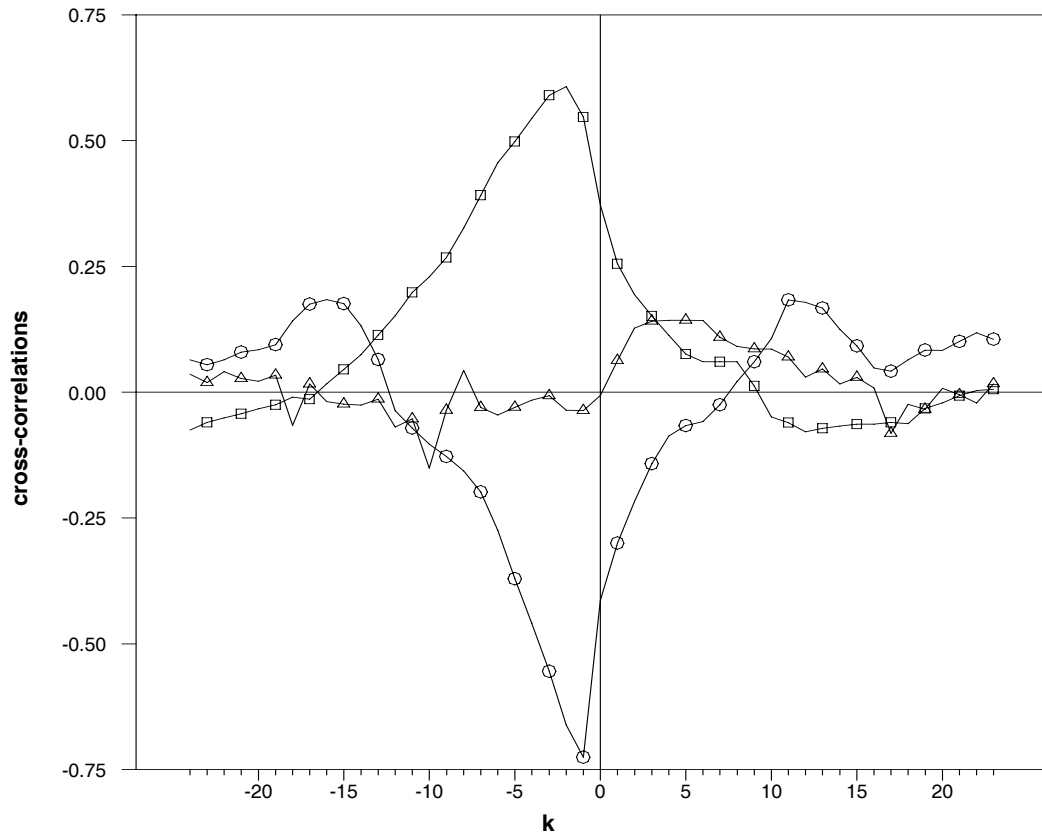
*Macroeconomic versus financial predictors, and monthly industrial production growth rates

Figure 3 – Model-based equity premium and economic conditions



*This picture is as Figure 1, but the equity premium is that based on the estimation of model (4)-(6). The predictive regression depicts the prediction of the Ordinary Least Squares: $\hat{\mathcal{E}} = 8.331 - \mathbb{I}_{\text{recession}} \cdot 2.038 \cdot \text{Ind} - \mathbb{I}_{\text{expansion}} \cdot 0.462 \cdot \text{Ind} + w$, where $\mathbb{I}_{\text{recession}}$ (resp. $\mathbb{I}_{\text{expansion}}$) is the indicator function taking the value one if the economy is in a NBER-recession (resp. expansion) episode and zero otherwise, w is a residual term, and standard errors are in parenthesis.

Figure 4 – Cross-correlations implied by model (4)-(6)*



*Cross-correlation between: risk-premium and industrial production growth k -months ahead (denoted with ○); risk-premium and returns volatility k -months ahead (denoted with □); returns and industrial production growth k -months ahead (denoted with △)